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SERVICEABILITY ANALYSIS OF FLEXURAL REINFORCED CONCRETE MEMBERS

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Abstract. The paper presents a simple discrete crack model for analyzing the deformation and crack width of reinforced concrete beams. The model is based on a non-iterative algorithm and uses a rigid-plastic bond-slip law and elastic properties of materials. Curvatures and crack widths predicted by the proposed technique were checked against the test results of six experimental beams, reported by the authors and other investigators. The article also proposes and discusses a numerical procedure for deriving the average bond stress with reference to the test data. Serviceability analysis resulted in a reasonable agreement on the test measurements.

Keywords: reinforced concrete, discrete cracking, bond stress-slip law, simplified model.

1. Introduction

Adequate modelling of reinforced concrete (RC) cracking, particularly post-cracking behaviour as one of the major sources of nonlinearity, is the most important and difficult task of serviceability analysis. Post-cracking deformation response of RC members is a process including a wide range of effects such as different strength and deformation properties of steel and concrete, the shrinkage and creep of concrete, bond slip between reinforcement and concrete. Due to this complexity, deflection predictions using different techniques may vary in the range of 20–37%, whereas the variability of predictions for crack width is of much higher order (Kaklauskas 2004; Gribniak 2009; Juozapaitis *et al.* 2010).

Fig. 1 presents a typical load-strain curve of RC members subjected to tension. The load-displacement diagram points to four stages of deformation behaviour. The first stage represents elastic deformations of the member up to the start of cracking (part OA). The second stage covers the region between the first and the final primary cracks (part AB). At the end of this stage (*Final cracking* point in Fig. 1), the RC member becomes separated by the developed cracks into a number of concrete blocks. The length of each block falls into specific interval $l_{tr} \le l_{cr} \le 2l_{tr}$, where l_{tr} is transfer length. It is reported (Bigaj 1999) that the average block length (crack spacing) could be in the range of $1.3l_{tr}$–$1.5l_{tr}$. The behaviour of the member with fully developed cracks (part BC) corresponds to the third stage. The fourth stage starts with the yielding of reinforcement. The problems of serviceability are mainly related to the third stage usually covering the service loading region. Similar stages of behaviour can be observed in bending members.

In the vicinity of cracks, reinforcement slip occurs and bond stress develops between the reinforcing bar and surrounding concrete, transmitting tensile force from the bar to concrete. Many theoretical models to predict deformations and/or the crack width of RC members have been proposed. Generally, the models may be divided into four groups:

- Semi-empirical: the earliest approaches were developed based on test data. Such simplified models are broadly presented in design codes but are not universal due to specific constitutive experiments;
- Fracture mechanics: such approaches use the principles of the fracture mechanics of concrete. They are generally applied in combination with other approaches analyzing RC structures;
- Average stress-average strain: simple approaches based on the smeared crack model. Such models are extensively used for numerical analysis and can evaluate the average response of a member; however, they are not able to predict the cracking character;
- Discrete crack: these approaches are suitable to assess the opening of each individual crack.

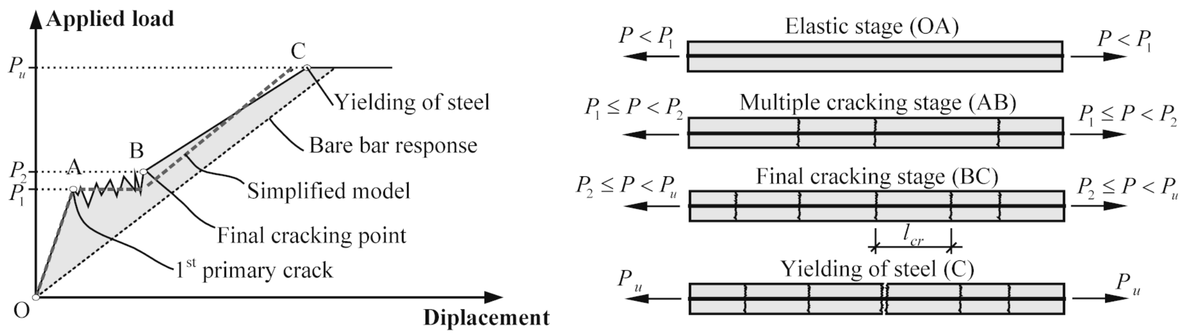

Fig. 1. Cracking stages of a tensile member

The latter models are capable of reflecting the realistic behaviour of RC members (Kwak, Ha 2006; Wu, Gilbert 2009; Ng *et al.* 2010); nevertheless, the prediction results become dependent on the assumed bond stress-slip law. Different bond-slip relationships were proposed by Eligehausen *et al*. (1983), Shima *et al*. (1987), Uijl den, Bigaj (1996) and other researchers. However, the application of these relationships is based on sophisticated calculation procedures. Marti *et al*. (1998) have suggested a simple model of RC tie assuming a rigid-plastic bond stress-slip law. This assumption resulted in idealized load-displacement behaviour as shown in Fig. 1 by the dashed line. This study extends such simplified approach for the serviceability analysis of RC flexural members.

2. Simplified discrete crack model

The model is based on the following assumptions:

1. Linear-elastic properties were assumed for reinforcement and concrete, both in tension and compression.
2. All cracks appear under cracking load dividing the beam into a number of blocks. Strain and stress distribution in a concrete block are symmetrical about its centre (Fig. 2).
3. Transfer length l_{tr} is calculated using the chosen technique. This study has assumed (EN 1992-1-1:2004) that $l_{tr} = S_{r,\max} / 2$ where $S_{r,\max}$ is maximum crack spacing. For analyzing deformation and average crack width, block length (crack spacing) l_{cr} is taken $1.5l_{tr}$, whereas $2l_{tr}$ is used for calculating maximum crack width.
4. Linear strain distribution is assumed for reinforcement both in tension and compression and compressive concrete. As shown in Figs. 3a and 3b, strain distribution in tensile concrete is also linear, but may have an individual shape (Fantilli *et al*. 1998; Kobielak *et al*. 2010).
5. Tensile concrete response in a cracked section is ignored.
6. Strain distribution of tensile reinforcement is assumed to be linear in a concrete block (Kankam 2003).

Strain in tensile reinforcement is related to bond stress by the following equation:

$$\frac{d\varepsilon_s}{dx} = \frac{4}{D_s E_s}\tau(x), \tag{1}$$

where D_s and E_s are the diameter and modulus of the elasticity of the steel bar respectively. Based on assumption 6 and relationship (1), we obtain $\tau(x)$ = const.

The equilibrium conditions of all forces and moments in the section result in the following equations:

$$\begin{aligned} N_{cc} + N_{sc} + N_{ct} + N_{st} &= 0; \\ M_{cc} + M_{sc} + M_{ct} + M_{st} &= M_{ext}, \end{aligned} \tag{2}$$

where M_{ext} is the external bending moment; N and M are internal forces and moments respectively. The first subscript corresponds to either c for concrete or s for steel and the second subscript refers to compression (c) or tension (t) as shown in Fig. 3d.

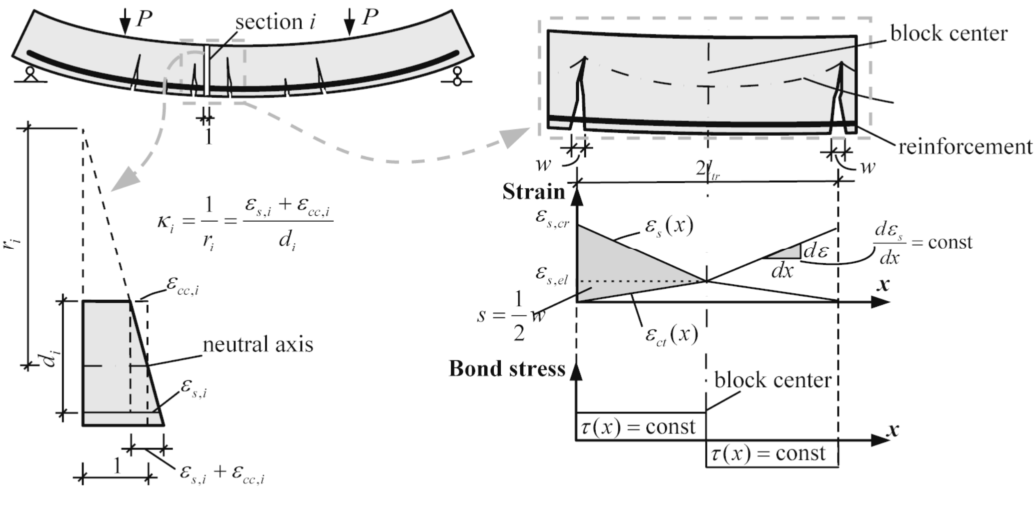

Fig. 2. Discrete crack approach in modelling a bending member

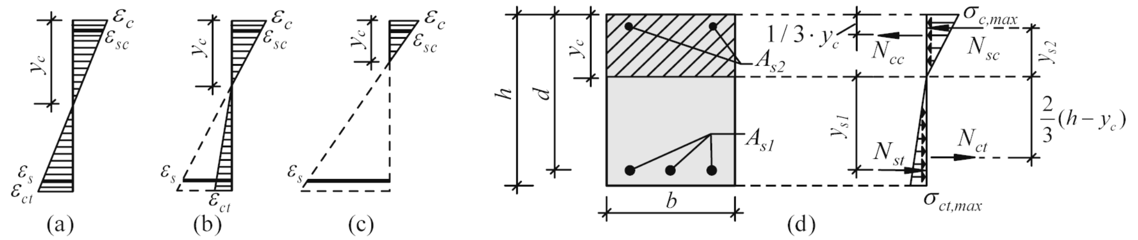

Fig. 3. Modelling a bending member: (a), (b), (c) assumed strain distribution in uncracked, arbitrary and cracked sections respectively; (d) internal forces

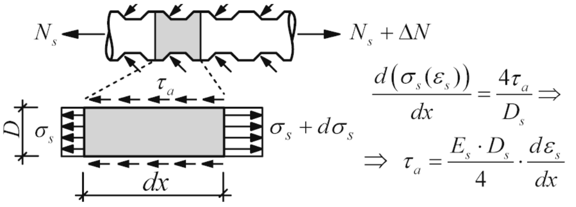

Fig. 4. Force equilibrium of a reinforcing bar

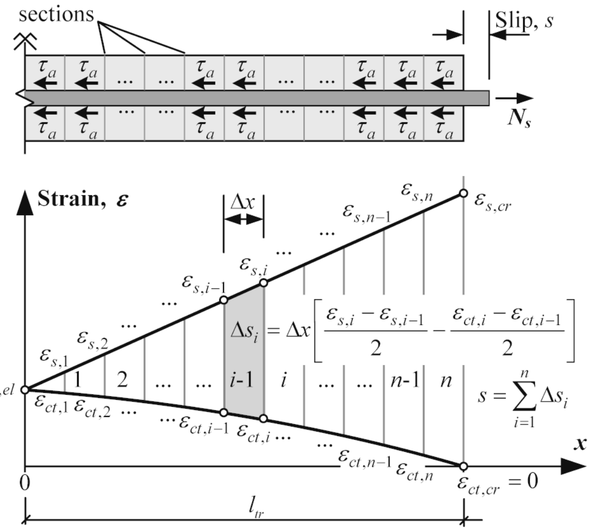

Fig. 5. Strain and bond-slip comparability conditions for the discrete cracking approach

The curvature and crack width of the RC member can be calculated by the following algorithm:

1. Calculate the second moments of the inertia of elastic and fully cracked sections I_{el}, I_{cr}.
2. Calculate the cracking moment:

$$M_{cr} = f_{ct} W_0 , \tag{3}$$

where f_{ct} is the strength of concrete in tension; W_0 is the elastic section modulus taken in respect to the tension edge of the transformed section.

3. At cracking load M_{cr}, calculate steel strains $\varepsilon_{s,cr}$ and $\varepsilon_{s,el}$ for fully cracked and uncracked sections respectively (see Fig. 2).
4. Define transfer length l_{tr} by EN 1992-1-1:2004 (2004) (see assumption 3).
5. Determine the strain distribution law of tensile reinforcement (see Fig. 2):

$$\varepsilon_s(x) = \varepsilon_{s,cr} - x \cdot \left(\varepsilon_{s,cr} - \varepsilon_{s,el}\right) / l_{tr} . \tag{4}$$

6. Based on equation (1) and Fig. 4, for cracking load M_{cr}, calculate bond stress τ_a. The latter value is applied at all successive load stages.
7. Divide the concrete block into a number of sections n as shown in Fig. 5 and calculate strain distribution along the block using the results of step 6 and equilibrium equations (2).
8. Calculate curvature at each section κ_i ($i = 1 \ldots n$) as shown in Fig. 2 where $\varepsilon_{s,i}$ and $\varepsilon_{cc,i}$ are steel and compressive concrete strain respectively.
9. Calculate the average curvature of the member:

$$\kappa = \frac{1}{n} \sum_{i=1}^{n} \kappa_i . \tag{5}$$

10. Calculate crack width w using the strains of steel $\varepsilon_s(x)$ and tensile concrete $\varepsilon_{ct}(x)$ obtained from Fig. 5:

$$w = 2 \cdot s = 2 \int_0^{l_{tr}} \left[\varepsilon_s(x) - \varepsilon_{ct}(x) \right] dx . \tag{6}$$

3. Modelling flexural members: a comparison with test results

The discussed model has been applied to simulate RC beams tested by Clark and Speirs (1978) and Kaklauskas *et al.* (2008). Fig. 6 shows the experimental and calculated moment-curvature diagrams for six RC sections with a different ratio of tensile reinforcement ρ. Beams *S2-3* and *S2-3R* were twin specimens, but had different reinforcement in the compression zone (2Ø6 and 3Ø14 respectively). The assumed rigid-plastic bond laws are shown in Fig. 7 using solid lines. Bond stresses τ_a ranged from $1.1f_{ct}$ to $1.8f_{ct}$ with the tendency that the beams with a lower reinforcement ratio and fewer bars possessed higher stresses. It should be noted that for tie members Marti *et al.* (1998) have assumed $\tau_a = 2f_{ct}$. The present study has shown that such assumption might result in unrealistically small crack width and spacing.

The accuracy analysis of curvature predictions at service load $M_{ser} = 0.6M_u$ (M_u is the ultimate moment) was performed. The obtained results are given in Table 1. The differences between the measured and calculated curvatures ranged from 2 to 5%, whereas, the average crack widths for beams *S2-2*, *S2-3* and *S2-3R* (Kaklauskas *et al.* 2008) were overestimated by about 30% (see Table 1). Such agreement can be considered as satisfactory: crack width analysis dealing with a single section, due to a stochastic nature of cracking, generally results in

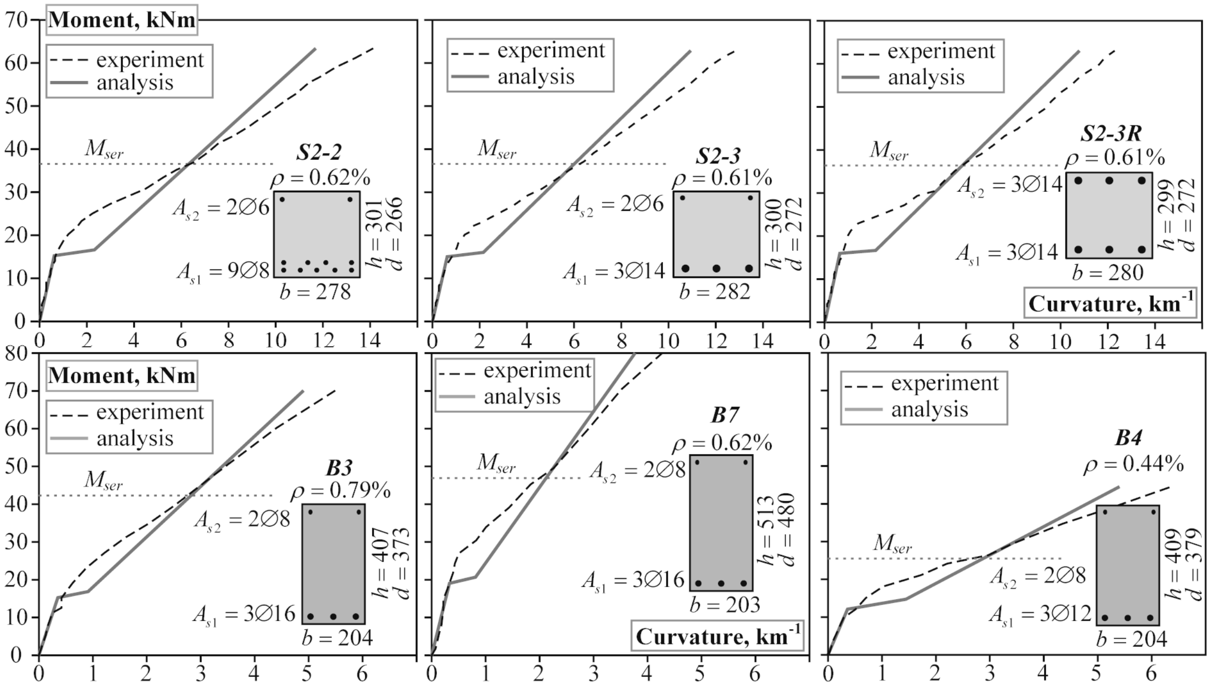

Fig. 6. Moment-curvature diagrams obtained by Kaklauskas *et al.* (2008) (top) and by Clark and Speirs (1978) (bottom)

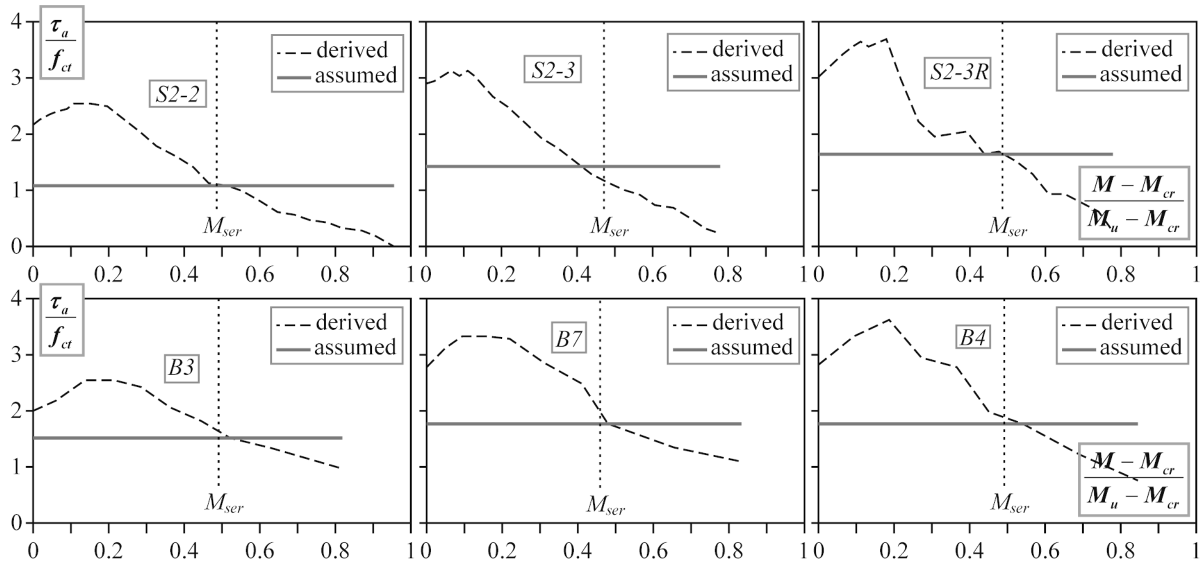

Fig. 7. Derived and assumed bond stresses

Table 1. The curvatures and average crack widths of the beams at service load

Beam	ρ [%]	τ_a / f_{ct}	κ_{exp} [km^{-1}]	κ_{an} [km^{-1}]	$\kappa_{an} / \kappa_{exp} - 1$ [%]	w_{exp} [mm]	w_{an} [mm]	$w_{an} / w_{exp} - 1$ [%]
B3	0.79	1.54	3.12	3.22	3.2	–	–	–
B7	0.62	1.76	2.96	3.04	2.7	–	–	–
B4	0.44	1.78	2.09	2.15	2.9	–	–	–
S2-2	0.62	1.08	6.59	6.46	–2.0	0.11	0.145	31.8
S2-3	0.61	1.43	6.40	6.09	–4.8	0.12	0.151	25.8
S2-3R	0.61	1.49	6.06	6.01	–0.8	0.12	0.147	22.5

larger errors in regard to deflections. The latter represent the global response of structure with smeared out local effects such as the cracking, slippage and degradation of bond stresses.

Numerous studies have shown that bond-stresses degrade with an increase in load (Torres *et al*. 2004; Kaklauskas *et al*. 2009, 2011a, b; Wu, Gilbert 2009; Gribniak *et al*. 2010; Kala *et al*. 2010; Ng *et al*. 2010; Zanuy 2010; Ho, Peng 2011). This also can be observed from the moment-curvature diagrams shown in Fig. 6. The modelled curvature response was too stiff for all beams due to the assumption of constant bond stress.

To overcome the above deficiency, the authors have developed an inverse procedure for deriving a rigid-plastic bond-slip law based on the test curvatures of RC beams. The concept of the procedure is similar to that described in Kaklauskas and Gribniak (2011). Fig. 7 shows the relationships between bond stress τ_a and the bending moment. The calculated and experimental moment-curvature diagrams will coincide at each loading level assuming respective bond stress. It should be noted that bond stresses calculated by the inverse technique were in a good agreement at service load with τ_a assumed in the discrete crack model (see Fig. 7 and Table 1).

4. Conclusions

The paper discusses the discrete crack model based on bond stress-slip relationship and considers its applicability for deformation and crack widths analyses of reinforced concrete flexural members. The performed serviceability analysis has indicated that the model becomes a useful tool for predicting crack widths and deformations. Based on the obtained results, the following conclusions can be drawn:

1. The simplified model based on the rigid-plastic bond stress-slip law was able to predict curvature and crack width. Inaccuracies in deflection predictions at service load varied from 2 to 5%. A satisfactory agreement was achieved for crack widths.

2. Bond stresses τ_a assumed in the discrete crack model ranged from $1.1f_{ct}$ to $1.8f_{ct}$ with the tendency that the beams with a lower reinforcement ratio and fewer bars possessed higher stresses. The application of $\tau_a = 2f_{ct}$, commonly taken for tie members, may result in underestimating crack width.

3. The potentials of the discussed technique should be further investigated regarding application for the modern types of reinforcement such as fibers and/or non-metallic bars. Moreover, the influence of various cases of distribution reinforcement bars in a section on cracking behaviour should be investigated.

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LENKIAMŲJŲ GELŽBETONINIŲ ELEMENTŲ TINKAMUMO ANALIZĖ

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S a n t r a u k a

Straipsnyje pateikiamas supaprastintas diskrečiųjų plyšių modelis gelžbetoninių sijų deformacijų ir plyšio pločio analizei. Modelis pagrįstas neiteraciniu algoritmu, remiantis standžiai plastiniu sukibimo dėsniu ir tampriosiomis medžiagų savybėmis. Remiantis skirtingų autorių atliktais eksperimentiniais duomenimis, modelis patikrintas skaičiuojant gelžbetoninių sijų kreivius ir plyšio pločius. Apskaičiuotos kreivių ir plyšio pločių reikšmės gana tiksliai sutapo su eksperimentiniais rezultatais. Taip pat pasiūlytas originalus vidutinių sukibimo įtempių apskaičiavimo metodas, remiantis eksperimentiniais lenkiamųjų gelžbetoninių sijų bandymo rezultatais.

Reikšminiai žodžiai: gelžbetonis, pleišėjimas, armatūros sukibimo įtempių ir praslydimo priklausomybė, supaprastintas modelis.

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