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Numerical Modelling of Cooperative and Noncooperative Three Transboundary Pollution Problems under Learning by Doing in Three Gorges Reservoir Area

Zuliang Lu[a,b,c], Lin Li[c], Longzhou Cao[c] and Yin Yang[d]

[a] *Key Laboratory for Nonlinear Science and System Structure, Chongqing Three Gorges University*
Chongqing 404000, China
[b] *Research Center for Mathematics and Economics, Tianjin University of Finance and Economics*
Tianjin, 300222, China
[c] *Key Laboratory of Intelligent Information Processing and Control, Chongqing Three Gorges University*
Chongqing 404000, China
[d] *Hunan Key Laboratory for Computation and Simulation in Science and Engineering, Key Laboratory of Intelligent Computing and Information Processing of Ministry of Education, Xiangtan University*
Xiangtan, 411105, Hunan, China
E-mail(*corresp.*): `zulianglux@126.com`
E-mail: `linligx@126.com`
E-mail: `caolongzhou@126.com`
E-mail: `yangyinxtu@xtu.edu.cn`

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Abstract. In this paper, we investigate cooperative and noncooperative three transboundary pollution problems in Three Gorges Reservoir Area where emission permits trading and abatement costs under learning by doing are considered. The abatement cost depends on two key factors: the level of pollution abatement and the experience of using pollution abatement technology. We use the optimal control theory to study the optimal emission paths and the optimal pollution abatement strategies under cooperative and noncooperative three transboundary pollution problems, respectively. By using the actual economic data of Wanzhou District, Kaizhou District and Yunyang County, we obtain the abatement level and the pollution stock of cooperative

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and noncooperative three transboundary pollution problems based on the four order Runge-Kutta method. We also discuss the influence of the change of parameter μ_i $(i = 1, 2)$ for the abatement level and the pollution stock.

Keywords: three transboundary pollution problem, learning by doing, emission permits trading, Three Gorges Reservoir Area, four order Runge-Kutta method.

AMS Subject Classification: 49J20; 65N30.

1 Introduction

Transboundary pollution problems refer to the emissions released by an area can travel via air movement or water flow and affect others area that are hundreds or sometimes thousands of miles away. According to this, it is necessary to study three transboundary pollution problems. Now, more and more, experts and scholars begin to pay attention to transboundary pollution problems. Transboundary pollution can cause great damage to people's lives and property in the spreading areas. With the complete completion of the Three Gorges project, the comprehensive benefits of flood control, power generation, shipping, water replenishment and other comprehensive benefits of the Three Gorges Dam project have been fully brought into play, but we should also pay attention to transboundary pollution problems in Three Gorges Reservoir Area. Transboundary pollution affects people's lives in the lower reaches of the Yangtze River. If this problem can not be effectively solved, it will not only endanger the lives and property safety of the people in the Three Gorges Reservoir Area and surrounding areas, but also hinder the sustainable development of the middle and lower reaches of the Yangtze River and even the whole country.

A mainly work of the Three Gorges Dam is to control flooding, which is a major problem for the seasonal river of the Yangtze. However, millions of people live in Yangtze River Basin, with many large and important cities like Chongqing City, Wuhan City, and Shanghai City situated adjacent to the river. Wanzhou District, Kaizhou District and Yunyang County, beside the Yangtze River, are located in Three Gorges Reservoir Area of Chongqing City. After storage of Three Gorges Reservoir Area, the problem of water security was concerned by the people of all circles. So, we must find a effect way to deal with water and air environmental transboundary pollution in the reservoir area and prevent the occurrence of a major pollution incident.

Recently, some published studies about transboundary pollution problems have been made from the aspect of renewable resources, clean technologies and domestic law, abatement cost and so on (for instance, see [4, 9, 20]). In 1992, Kaitala et al. [12] studied some different ways to deal with a dynamic game of transboundary air pollution between Finland and the nearby areas of the Soviet Union under both cooperative and noncooperative behaviors. In 2000, List et al. [16] discussed a general transboundary pollution without focusing on a specific type. They introduced the idea of a pollution stock that is directly affected by the polluter's emissions. In 2007, Yeung presented a cooperative differential game model of transboundary industrial pollution in [22]. In [23], the authors investigated a cooperative stochastic differential game of transboundary industrial pollution and a payment distribution mechanism to maintain the subgame consistency. Based on Yeung's model, Li [13] took emission permits trading into the game, in which the revenues were influenced by emission permits prices and initial quotas. Chang et al. [8] extended Li's work to a stochastic version and presented a numerical method to solve the model. Above all literatures about transboundary pollution game took emission permits trading and abatement into consideration, and the two mechanisms affected the optimal strategies to some extent. Furthermore, Li [14] investigated the relationship between the emission permits and the abatement investment by presenting an optimal control model. Li and Pan [15] constructed a dynamic general equilibrium model of pollution to derive the steady-state equilibrium properties and optimal levels of emission permits and pollution treatment. Similar to [8], we have used the fitted finite volume method to study three transboundary pollution of Three Gorges Reservoir Area with the emission permits trading by cooperative stochastic differential game in [17]. To improve the emission reduction and accumulate the emission reduction experience, the thought of learning by doing has been more and more used in the water pollution emissions trading, and this idea has also been widely used in several other industries, such as clothing manufacturing, instrument manufacturing, automotive assembly, semiconductor manufacturing, has attracted a lot of scholars [2, 3, 10].

In this work, we study cooperative and noncooperative three transboundary pollution problems in the Three Gorges Reservoir Area where emission permits trading and abatement costs under learning by doing are considered. In [7], Chang et al. presented a transboundary pollution game which emission permits trading and abatement costs under learning by doing. Based on this, we extend this model to a three regions and present four order Runge-Kutta method to solve them. We obtain the optimal emission paths and optimal abatement levels of Wanzhou District, Kaizhou District and Yunyang County in Three Gorges Reservoir Area. This way of measuring the value of learning has been widely used in the operations research literature which takes learning by doing into account, such as [6, 11, 15, 21]. The emission permits trading scheme is also presented to keep up with the latest research advance, and there are some studies have mentioned to the environmental policy and abatement cost under learning by doing in [1, 6, 18, 19]. Therefore, it is great significance and application prospect to study cooperative and noncooperative three transboundary pollution problems under learning by doing in Three Gorges Reservoir Area.

The rest of this paper is organized as follows. In Section 2, we will establish a basic dynamic general equilibrium model of three transboundary pollution problems in Three Gorges Reservoir Area. The optimal emission paths and optimal abatement levels for the cooperative three transboundary pollution problems are presented in Section 3. Furthermore, we discuss the noncooperative three transboundary pollution problems, and it's optimal emission paths and optimal abatement levels in the Section 4. Finally, we will discuss the effects of parameters for cooperative and noncooperative three transboundary pollution problems of Three Gorges Reservoir Area by using four order Runge-Kutta method in Section 5.

2 The basic model

In this section, we will discuss a basic dynamic general equilibrium model of three transboundary pollution problems in Three Gorges Reservoir Area. For region i $(i = 1, 2, 3)$, production always leads to a quantity of by-products, namely emissions $E_i(t)$. We assume that $R_i(E_i(t))$ represents the production revenue value at time t, which can be expressed by the following quadratic functional form in terms of emissions:

$$R_i(E_i(t)) = A_i E_i(t) - 0.5E_i^2(t), \tag{2.1}$$

where A_i $(i = 1, 2, 3)$ is a positive constant. The above function can guarantee that the marginal production value revenue is decreasing. According to [8], we set $A_2 = \alpha_1 A_1$, and $A_3 = \alpha_2 A_2$, where α_1 and α_2 are two positive constants and they measure the gap between the three players' ability in obtaining benefit from production.

As to the emission permits trading, we first denote the price of emission permits by S and the positive initial emission quota by E_{i0}. Then, we can know that the emission permits revenue $Q_i(E_i(t))$ at time t should be

$$Q_i(E_i(t)) = S(E_i(t) - E_{i0}), \tag{2.2}$$

where $Q_i(E_i(t)) > 0$ $(i = 1, 2, 3)$ means that region i needs to purchase the emission permits from markets, $Q_i(E_i(t)) < 0$ means that region i can gain benefit by selling unused emission permits to others.

Moreover, we use $P(t)$ to stand for the pollution stock in the environment at time t. With the progress of technology and improved awareness of abatement, more and more countries are willing to spend money and time in controlling the pollution in the environment. Following [5], the dynamic process of pollution stock is governed by the following ordinary differential equation:

$$\frac{dP}{dt} = \sum_{i=1}^{3}(E_i(t) - a_i(t)) - \theta P(t), \quad P(0) = P_0, \quad P(t) > 0,$$

where $E_i(t)$ $(i = 1, 2, 3)$ represents the emission level of the region i, θ represents the exponential decay rate of pollution, and $a_i(t)$ is the abatement level at time t with the initial value $a_i(0) = a_0$. According to [8], the damage caused by the stock of pollution can be measured by $D_i P_i(t)$ for region i at time t, where D_i $(i = 1, 2, 3)$ is a positive parameter. Without loss of generality, we assume $D_2 = \beta_1 D_1$, and $D_3 = \beta_2 D_2$ where β_1 and β_2 are two positive constants and they measure the gap between the three players' ability in suffering damages from the pollution stock.

It is known that pollution abatement can be realized only when technique and labor are invested. So, we should face the abatement cost which could decrease the net revenue. Here we assume that the abatement cost can be described by following the quadratic form $\frac{1}{2}C_i a_i^2(t)$, where C_i are positive constants. We set $C_2 = \eta_1 C_1$, and $C_3 = \eta_2 C_2$, in which η_1 and η_2 are two positive parameters and they measure the difference between the three regions' ability in mastering the abatement technology (the better it masters the abatement technology, the fewer costs of the abatement will be). This form means that the marginal cost is increasing with respect to the level of pollution abatement. By means of [6], the experience of applying pollution abatement technology $Z(t)$ is measured by the cumulative abatement from time 0 to t, that is

$$Z_i(t) = Z_{0i} + b_i \int_0^t a_i(s)ds, \quad Z_i(0) = Z_{0i},$$

where Z_0 denotes the initial experience level of applying pollution abatement technology. Similar to the above, we set $b_2 = \mu_1 b_1$, and $b_3 = \mu_2 b_2$ in which μ_1 and μ_2 are two positive parameters and they represent the differences between the three regions' ability in accumulating experience. According to the learning by doing theory, the amount of cumulative experience will lead to a decline in the unit cost.

Furthermore, the current goal of region i is to maximize the expected present flow of instantaneous net revenue in terms of the emission path and the abatement level. Hence, the objective functional can be given as follows:

$$\max_{E_i(t),a_i(t)\ (i=1,2,3)} \int_0^\infty e^{-rt}\big[R_i(E_i(t)) + Q_i(E_i(t)) - D_iP(t) \\ - 0.5C_ia_i^2(t) + (Z_i(t) - Z_{0i})\big]dt.$$

Substituting (2.1) and (2.2) into the above model, we obtain the objective functional and the constraint conditions of region i $(i = 1, 2, 3)$ as follows:

$$\max_{E_i(t),a_i(t)} \textstyle\int_0^\infty e^{-rt}\big[(A_i - S)E_i(t) - \frac{1}{2}E_i^2(t) + SE_{i0} - D_iP(t) \\ -\frac{1}{2}C_ia_i^2(t) + b_i \int_0^t a_i(s)ds\big]dt,$$
$$\text{s.t.} \quad \frac{dP}{dt} = \sum_{i=1}^{3}(E_i(t) - a_i(t)) - \theta P(t), \quad P(0) = P_0.$$

3 Cooperative three transboundary pollution problems

In game theory, a cooperative game is a game with competition between players due to the possibility of external enforcement of cooperative behavior. In our model, three regions should follow the rules made by the legal agreements, and the joint optimal goal will be achieved. Their joint objective functional and constraint conditions can be described as follows:

$$\max_{E_{Ci}(t),a_{Ci}(t)\ (i=1,2,3)} \int_0^\infty e^{-rt}\left[\sum_{i=1}^{3}(A_i - S)E_{Ci}(t) - \sum_{i=1}^{3}\frac{E_{Ci}^2(t)}{2} + S\sum_{i=1}^{3}E_{i0} \\ -\sum_{i=1}^{3}D_iP(t) - \frac{1}{2}\sum_{i=1}^{3}(C_ia_{Ci}^2(t)) + \sum_{i=1}^{3}\left(b_i\int_0^t a_{Ci}(s)ds\right)\right]dt,$$
$$\text{s.t.} \quad \frac{dP(t)}{dt} = \sum_{i=1}^{3}(E_{Ci}(t) - a_i(t)) - \theta P(t), \quad P(0) = P_0,$$

where $E_{C1}(t)$, $E_{C2}(t)$, and $E_{C3}(t)$ denote the emission levels of regions 1–3, $a_{C1}(t)$, $a_{C2}(t)$, and $a_{C3}(t)$ are the abatement levels of regions 1–3 under the cooperative framework. Then, by using use Pontryagin's maximum principle, we can get the current value Hamiltonian for this optimal control problem:

$$H=\sum_{i=1}^{3}(A_i-S)E_{Ci}(t)-\sum_{i=1}^{3}\frac{E_{Ci}^2(t)}{2}+S\sum_{i=1}^{3}E_{i0}-\sum_{i=1}^{3}D_iP(t)-\frac{1}{2}\sum_{i=1}^{3}C_ia_{Ci}^2(t)$$
$$+\sum_{i=1}^{3}\left(b_i\int_0^t a_{Ci}(s)ds\right)+\lambda(t)\left(\sum_{i=1}^{3}(E_{Ci}(t)-a_i(t))-\theta P(t)\right),$$

where $\lambda(t)$ is the dynamic adjoint variable associated with its respective state equation for $\frac{dP(t)}{dt}$ in the cooperative game. The shadow price $\lambda(t)$ is the Lagrange multiplier, which is the derivative of the joint revenue with respect to the pollution stock P. The economic interpretation of $\lambda(t)$ refers to the impact of adding an additional unit pollution stock on the joint future profits. Moreover, $\lambda(t)>0$ means that the association benefits from the current pollution stock by lowering emission levels and increasing abatement level, which sacrifices the current profits for future profits, and vice versa. In addition, the Hamiltonian H also represents the instantaneous equilibrium condition, and the necessary condition for solving the optimal control problem is that the control variables are chosen to maximize H.

Therefore, we can get the necessary conditions from the following equations:

$$\frac{\partial H}{\partial E_{C1}(t)}=A_1-S-E_{C1}(t)+\lambda(t)=0, \tag{3.1}$$
$$\frac{\partial H}{\partial a_{C1}(t)}=-C_1a_{C1}+\frac{b_1a_{C1}(t)}{\frac{da_{C1}(t)}{dt}}-\lambda(t)=0, \tag{3.2}$$
$$\frac{\partial H}{\partial E_{C2}(t)}=A_2-S-E_{C2}(t)+\lambda(t)=0, \tag{3.3}$$
$$\frac{\partial H}{\partial a_{C2}(t)}=-C_2a_{C2}(t)+\frac{b_2a_{C2}(t)}{\frac{da_{C2}(t)}{dt}}-\lambda(t)=0, \tag{3.4}$$
$$\frac{\partial H}{\partial E_{C3}(t)}=A_3-S-E_{C3}(t)+\lambda(t)=0, \tag{3.5}$$
$$\frac{\partial H}{\partial a_{C3}(t)}=-C_3a_{C3}(t)+\frac{b_3a_{C3}(t)}{\frac{da_{C3}(t)}{dt}}-\lambda(t)=0, \tag{3.6}$$
$$\frac{d\lambda(t)}{dt}=r\lambda(t)-\frac{\partial H}{\partial P(t)}=(r+\theta)\lambda(t)+\sum_{i=1}^{3}D_i, \tag{3.7}$$
$$\frac{dP(t)}{dt}=\sum_{i=1}^{3}(E_{Ci}(t)-a_i(t))-\theta P(t). \tag{3.8}$$

According to the first-order optimality condition, we identify the optimal level of pollution abatement and optimal emission under steady state by the superscript "*". From equation (3.7) , we can obtain $\lambda(t)=\frac{-(D_1+D_2+D_3)}{r+\theta}$ according to the steady condition $\frac{d\lambda(t)}{dt} = 0$. Then substituting $\lambda(t)$ into (3.1)–(3.6), we have

$$\begin{aligned}
E^*_{C1}(t) =& A_1 - S - \frac{(D_1 + D_2 + D_3)}{r + \theta}, && (3.9)\\
E^*_{C2}(t) =& A_2 - S - \frac{(D_1 + D_2 + D_3)}{r + \theta}, &&\\
E^*_{C3}(t) =& A_3 - S - \frac{(D_1 + D_2 + D_3)}{r + \theta}, && (3.10)\\
\frac{da^*_{C1}(t)}{dt} =& \frac{b_1 a^*_{C1}(t)}{C_1 a^*_{C1}(t) - (D_1 + D_2 + D_3)/(r + \theta)}, && (3.11)\\
\frac{da^*_{C2}(t)}{dt} =& \frac{b_2 a^*_{C2}(t)}{C_2 a^*_{C2}(t) - (D_1 + D_2 + D_3)/(r + \theta)}, && (3.12)\\
\frac{da^*_{C3}(t)}{dt} =& \frac{b_3 a^*_{C3}(t)}{C_3 a^*_{C3}(t) - (D_1 + D_2 + D_3)(r + \theta)}. && (3.13)
\end{aligned}$$

Since the analytical solutions of (3.11), (3.12) and (3.13) cannot be gained, we can only solve them numerically. By using the four order Runge-Kutta method as follows:

$$\begin{cases}
y_{n+1} = y_n + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4),\\
K_1 = f(x_n, y_n), \quad K_2 = f(x_{\frac{n+1}{2}}, y_n + \frac{h}{2}K_1),\\
K_3 = f(x_{\frac{n+1}{2}}, y_n + \frac{h}{2}K_2), \quad K_4 = f(x_{n+1}, y_n + hK_3),
\end{cases}$$

where we choose $h = 0.1$. Then we can obtain the three regions' abatement levels of the cooperative three transboundary pollution problems.

4 Noncooperative three transboundary pollution problems

Each player tries his best to maximize his own net revenue by choosing the optimal emission path and the optimal abatement level under a noncooperative game. In our model, the objective functions of region 1, region 2 and region 3 are as follows:

$$\begin{aligned}
\max_{E_1(t), a_1(t)} \int_0^\infty & e^{-rt}\Big[(A_1 - S)E_1(t) - \frac{1}{2}E_1^2(t) + SE_{10} - D_1 P(t) - \frac{1}{2}C_1 a_1^2(t)\\
&+ b_1 \int_0^t a_1(s)ds\Big]dt,\\
\text{s.t.} \quad \frac{dP(t)}{dt} &= \sum_{i=1}^{3}(E_i(t) - a_i(t)) - \theta P(t), \quad P(0) = P_0,
\end{aligned}$$

and

$$\begin{aligned}
\max_{E_2(t), a_2(t)} \int_0^\infty & e^{-rt}\Big[(A_2 - S)E_2(t) - \frac{1}{2}E_2^2(t) + SE_{20} - D_2 P(t) - \frac{1}{2}C_2 a_2^2(t)\\
&+ b_2 \int_0^t a_2(s)ds\Big]dt,
\end{aligned}$$

$$\text{s.t.}\frac{dP(t)}{dt} = \sum_{i=1}^{3}(E_i(t) - a_i(t)) - \theta P(t), \quad P(0) = P_0,$$

and

$$\max_{E_3(t),a_3(t)} \int_0^{\infty} e^{-rt}\Big[(A_3 - S)E_3(t) - \frac{1}{2}E_3^2(t) + SE_{30} - D_3P(t) - \frac{1}{2}C_3a_3^2(t)$$
$$+ b_3\int_0^t a_3(s)ds\Big]dt,$$
$$\text{s.t.}\frac{dP(t)}{dt} = \sum_{i=1}^{3}(E_i(t) - a_i(t)) - \theta P(t), \quad P(0) = P_0.$$

In order to obtain the optimality conditions for the optimal control problems, we use the Pontryagin's maximum principle. The current value Hamiltonians for this optimal control problem are: where $\lambda_1(t)$, $\lambda_2(t)$ and $\lambda_3(t)$ are the dynamic adjoint variables associated with the state equation about $\frac{dP(t)}{dt}$. Here, the dual variables $\lambda_1(t)$, $\lambda_2(t)$ and $\lambda_3(t)$, also called shadow prices, are Lagrange multipliers, which are the derivatives of the three players' value functions, i.e. revenues, with respect to the pollution stock P. Economically, they refer to the impact of adding an additional unit pollution stock on the three players' future profits. A positive shadow price implies that the players benefit from the current pollution stock by lowering emission levels and increasing abatement level, which sacrifices the current profits for future profits, and vice versa. Furthermore, the Hamiltonians H_1, H_2 and H_3 represent the instantaneous equilibrium conditions, and a necessary condition for solving the above optimal control problem is that the emission levels (E_1, E_2, E_3) and the abatement levels (a_1, a_2, a_3) should be chosen to maximize the H_1, H_2 and H_3 respectively.

Next we derive the following necessary conditions:

$$\frac{\partial H_1}{\partial E_1(t)} = A_1 - S - E_1(t) + \lambda_1(t) = 0, \tag{4.1}$$

$$\frac{\partial H_1}{\partial a_1(t)} = - C_1a_1(t) + \frac{b_1a_1(t)}{\frac{da_1(t)}{dt}} - \lambda_1(t) = 0, \tag{4.2}$$

$$\frac{\partial H_1}{\partial P(t)} = - D_1 - \theta\lambda_1(t),$$

$$\frac{d\lambda_1(t)}{dt} = r\lambda_1(t) - \frac{\partial H_1}{\partial P(t)} = (r + \theta)\lambda_1(t) + D_1, \tag{4.3}$$

$$\frac{dP(t)}{dt} = \sum_{i=1}^{3}(E_i(t) - a_i(t)) - \theta P(t). \tag{4.4}$$

From (4.1), we have

$$\lambda_1(t) = S + E_1(t) - A. \tag{4.5}$$

Substituting (4.5) into (4.3), we obtain

$$\frac{d\lambda_1(t)}{dt} = (r + \theta)\lambda_1(t) + D_1 = (r + \theta)(S + E_1(t) - A_1) + D_1. \tag{4.6}$$

Now, we investigate a system of two variables described by the differential equations (4.4) and (4.6). By the definition, the steady state conditions should be $\frac{dP(t)}{dt} = \frac{d\lambda_1(t)}{dt} = 0$. Then from (4.4) and (4.6), we have

$$\begin{aligned} &E_1(t) - a_1(t) + E_2(t) - a_2(t) + E_3(t) - a_3(t) - \theta P(t) = 0, \\ &(r+\theta)(S + E_1(t) - A_1) + D_1 = 0. \end{aligned} \tag{4.7}$$

Similarly, for the current value Hamiltonian H_2 we obtain

$$\begin{aligned} &\frac{\partial H_2}{\partial E_2(t)} = A_2 - S - E_2(t) + \lambda_2(t) = 0, \\ &\frac{\partial H_2}{\partial a_2(t)} = -C_2 a_2(t) + \frac{b_2 a_2(t)}{\frac{da_2(t)}{dt}} - \lambda_2(t) = 0, \end{aligned} \tag{4.8}$$

$$\begin{aligned} &\frac{\partial H_2}{\partial P(t)} = -D_2 - \theta\lambda_2(t), \\ &\frac{d\lambda_2(t)}{dt} = r\lambda_2(t) - \frac{\partial H_2}{\partial P(t)} = (r+\theta)\lambda_2(t) + D_2, \\ &(r+\theta)(S + E_2(t) - A_2) + D_2 = 0. \end{aligned} \tag{4.9}$$

Similarly, for the current value Hamiltonian H_3 we obtain

$$\begin{aligned} &\frac{\partial H_3}{\partial E_3(t)} = A_3 - S - E_3(t) + \lambda_3(t) = 0, \\ &\frac{\partial H_3}{\partial a_3(t)} = -C_3 a_3(t) + \frac{b_3 a_3(t)}{\frac{da_3(t)}{dt}} - \lambda_3(t) = 0, \\ &\frac{\partial H_3}{\partial P(t)} = -D_3 - \theta\lambda_3(t), \\ &\frac{d\lambda_3(t)}{dt} = r\lambda_3(t) - \frac{\partial H_3}{\partial P(t)} = (r+\theta)\lambda_3(t) + D_3, \\ &(r+\theta)(S + E_3(t) - A_3) + D_3 = 0. \end{aligned} \tag{4.10}$$

Thus, we can obtain the following results from (4.7) and (4.9):

$$\lambda_1(t) = \frac{-D_1}{r+\theta}, \quad \lambda_2(t) = \frac{-D_2}{r+\theta}, \quad \lambda_3(t) = \frac{-D_3}{r+\theta}. \tag{4.11}$$

According to the first-order optimality condition, we identify the optimal level of pollution abatement and optimal emission under steady state by the superscript "*". Therefore, substituting (4.11) into (4.2), (4.8) and (4.10), respectively, we can obtain that

$$E_1^*(t) = A_1 - S - \frac{D_1}{r+\theta}, \quad E_2^*(t) = A_2 - S - \frac{D_2}{r+\theta}, \tag{4.12}$$

$$E_3^*(t) = A_3 - S - \frac{D_3}{r+\theta}, \quad \frac{da_1^*(t)}{dt} = \frac{b_1 a_1^*(t)}{C_1 a_1^*(t) - \frac{D_1}{r+\theta}}, \tag{4.13}$$

$$\frac{da_2^*(t)}{dt} = \frac{b_2 a_2^*(t)}{C_2 a_2^*(t) - \frac{D_2}{r+\theta}}, \quad \frac{da_3^*(t)}{dt} = \frac{b_3 a_3^*(t)}{C_3 a_3^*(t) - \frac{D_3}{r+\theta}}. \tag{4.14}$$

Similarly, the analytical solutions of remaining equations cannot be gained, thus we use the fourth order Runge-Kutta method to solve them.

5 Effects of parameters

In this section, we will discuss the effects of parameters for cooperative and non-cooperative three transboundary pollution problems of Three Gorges Reservoir Area by using four order Runge-Kutta method. Three Gorges Reservoir Area is the region involved in the submersion of the water storage of the reservoir region of the Three Gorges Dam, mainly including Wanzhou District, Kaizhou District, and Yunyang County in Chongqing City. Three Gorges Reservoir Area is located in the upstream of the Yangtze River with the area of 59900 km^2. Three Gorges Reservoir Area stretches along the Yangtze River from Jiangjin District of Chongqing City to Yichang City of Hubei Province, which is very narrow and where the geography is complex. The mountainous areas represent 74% of the region only with 4.3% plain area in the river valley and 21.7% hilly area. The climate of the reservoir region of the Three Gorges Project is the subtropical monsoon climate. Three Gorges abounds in the water resources, the ecological resources, the tourism resources and the resources of some ores, which has a great potential for exploration. The Yangtze River flows through Three Gorges Reservoir Region with the good conditions for water transportation and the water-resources development. The potential installed capacity of hydropower can be about 30000 MW in Three Gorges Reservoir Area where the water resources are the most abundant for the whole main stream of the Yangtze River. However, three transboundary pollution problem of Three Gorges Reservoir Area has became an important pollution problem in China. With the rapid development of economy, three transboundary industrial pollution on the impact of Three Gorges Reservoir Area have gradually increased.

Firstly, in Table 1 we give some economic data of Wanzhou District, Kaizhou District, and Yunyang County in Chongqing City from 2012 to 2016, where GDP denotes the gross domestic product (hundred million yuan), mean denotes

Table 1. The gross domestic product and economic growth rate of Wanzhou District, Kaizhou District, and Yunyang County.

year	Wanzhou District		Kaizhou District		Yunyang County	
	GDP	growth rates	GDP	growth rates		growth rates
2012	662.86	10.6	229.55	11.5	126.63	11.6
2013	702.03	12.5	265.47	13.8	150.34	12.5
2014	771.22	11.1	300.17	12.0	170.19	11.8
2015	828.22	11.1	325.98	11.6	187.91	11.1
2016	897.39	10.8	360.62	10.9	213.11	10.4
mean	776.97	11.2	299.86	12.0	172.24	11.5

the quadratic mean, for $n \in \mathbb{R}$ and a_i (a_i denotes GDP or growth rates),

$$\text{mean} = \sqrt{(a_1^2 + a_2^2 + \cdots + a_n^2)/n}.$$

In Wanzhou District, Kaizhou District and Yunyang County, carbon emission trading price are 0.1–0.3 Yuan/10kg, we set the permits price $S = 0.15$. Total carbon dioxide emissions of Wanzhou District, Kaizhou District, and Yunyang County are about 100–1000 million tons, we set the stock of pollution $P_0 = 500$. From the Table 1, the quadratic mean of gross domestic product are 776.97 hundred million yuan in Wanzhou District, 299.86 hundred million yuan in Kaizhou District, and 172.24 hundred million yuan in Yunyang County from 2012 to 2016, so we choose $\alpha_1 = 299.86/776.97 = 0.39$ and $\alpha_2 = 172.24/299.86 = 0.57$. In 2016, the gross domestic product are 897.39 hundred million yuan in Wanzhou District, 360.62 hundred million yuan in Kaizhou District, and 213.11 hundred million yuan in Yunyang County, we choose $\eta_1 = 897.39/360.62 = 2.49$ and $\eta_2 = 360.62/213.11 = 1.69$. The quadratic mean of growth rates are 11.2 percent in Wanzhou District, 12.0 percent in Kaizhou District, and 11.5 percent in Yunyang County from 2012 to 2016, we also choose $\mu_1 = 11.2/12 = 0.93$ and $\mu_2 = 12/11.5 = 1.04$. Similarly, we set $\beta_1 = 0.25$ and $\beta_2 = 0.46$. We assume that the positive constants $T = 10, A_1 = 5, D_1 = 0.1, a_0 = 5, \theta = 0.6, C_1 = 2, b_1 = 2, r = 0.08$, which similar to [8]. According to (3.9)–(3.10) and (4.12)–(4.13), we can obtain emission levels of the cooperative and noncooperative three transboundary pollution problems in Wanzhou District, Kaizhou District and Yunyang County as follows: $E_{C1}^* = 4.6493$, $E_{C2}^* = 1.5993$, $E_{C3}^* = 0.7608$, $E_1^* = 4.7029$, $E_2^* = 1.7632$, $E_3^* = 0.9446$.

The emission levels of the cooperative and noncooperative three transboundary pollution problems in Wanzhou District, Kaizhou District and Yunyang County are plotted in Figure 1.

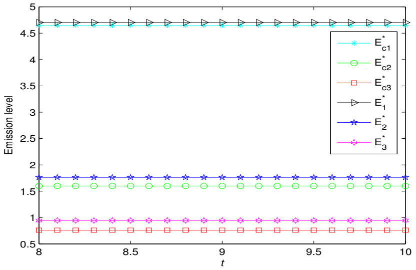

Figure 1. The emission levels of the cooperative and noncooperative transboundary pollution problems in Wanzhou District, Kaizhou District and Yunyang County.

Furthermore, from (3.11), (3.12) and (3.13), by using the Runge-Kutta method to solve them, we can get abatement levels of the cooperative transboundary pollution problems in Wanzhou District, Kaizhou District and Yunyang County, and show in Figure 2.

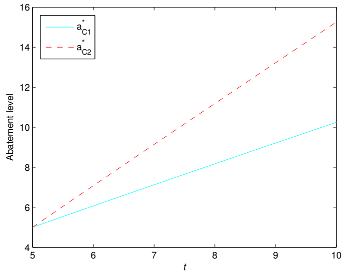

Figure 2. The abatement levels of the cooperative transboundary pollution problems in Wanzhou District, Kaizhou District and Yunyang County.

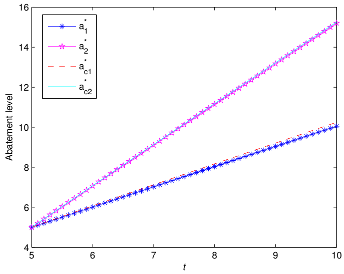

Figure 3. The abatement levels of the noncooperative transboundary pollution problems in Wanzhou District, Kaizhou District and Yunyang County.

Similarly, by using (4.13), (4.14) we can obtain abatement levels of the noncooperative transboundary pollution problems in Wanzhou District, Kaizhou District and Yunyang County are plotted in Figure 3.

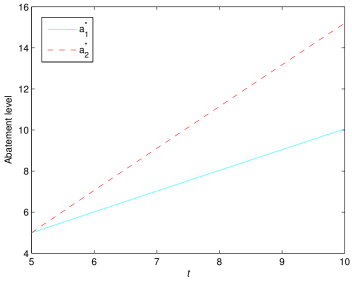

Figure 4. Comparison emission levels and abatement levels between cooperative and noncooperative transboundary pollution problems.

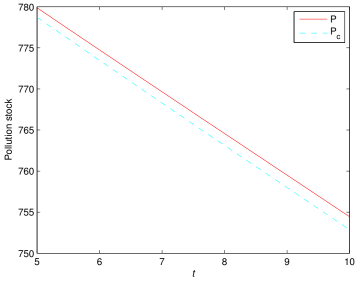

Figure 5. Comparison pollution stocks between cooperative and noncooperative transboundary pollution problems.

As shown in Figures 2–4, we can clearly see that the abatement levels of Wanzhou District, Kaizhou District and Yunyang County under the cooperative and noncooperative transboundary pollution problems improve with the increasing of t, and the abatement levels of Wanzhou District increased the fastest in the same time, Yunyang County the slowest. Abatement level-time ratio of Wanzhou District rose sharply, but Kaizhou District and Yunyang County rose gently. Then, from Figures 4–5, we also can clearly see that the effects of the abatement levels of Wanzhou District, Kaizhou District and Yunyang County under cooperative transboundary pollution problems is similar to that the abatement levels of Wanzhou District, Kaizhou District and Yunyang County under noncooperative transboundary pollution problems, it is means that we can use two ways to deal with the abatement levels of Wanzhou District, Kaizhou District and Yunyang County. But the pollution stock under

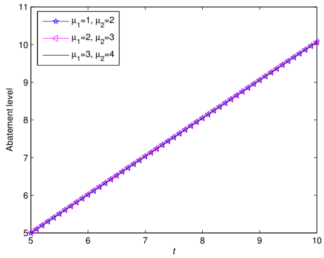

Figure 6. The effects of μ_i $(i = 1, 2)$ on Wanzhou District's abatement levels of cooperative transboundary pollution problems.

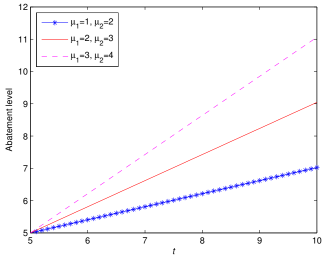

Figure 7. The effects of μ_i $(i = 1, 2)$ on Kaizhou District's abatement levels of cooperative transboundary pollution problems.

the cooperative transboundary pollution problems is lower than that under the noncooperative transboundary pollution problems.

Now, we will analyze the effects of the Wanzhou District, Kaizhou District and Yunyang County's abatement levels and pollution stocks with different values of parameter μ_i $(i = 1, 2)$. From the expressions of emission levels and Figure 1, we can clearly see that there is no relationship between E_i^* and μ_i $(i = 1, 2)$. Hence, we do not discuss the effects of μ_i $(i = 1, 2)$ on the emission levels. We set $\mu_1 = 1, 2, 3$ and $\mu_2 = 2, 3, 4$. Then, the effects of parameter μ_i $(i = 1, 2)$ on the cooperative three transboundary pollution problems in Wanzhou District, Kaizhou District and Yunyang County's abatement levels and pollution stocks have been shown in Figures 6–9.

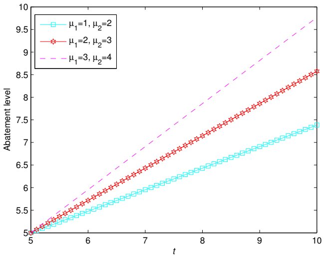

Figure 8. The effects of μ_i $(i = 1, 2)$ on Yunyang County's abatement levels of cooperative transboundary pollution problems.

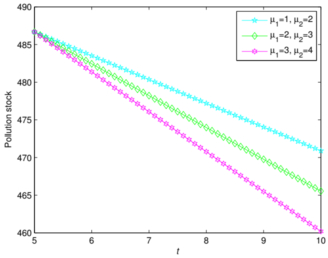

Figure 9. The effects of μ_i $(i = 1, 2)$ on pollution stock of cooperative transboundary pollution problems.

As shown in Figures 6–8, it can clearly see that there is no influence of the Wanzhou District's abatement levels with the increasing of μ_i $(i = 1, 2)$, and the abatement levels of Kaizhou District and Yunyang County will improve with the increasing of μ_i $(i = 1, 2)$. In other words, by using the definition of μ_i $(i = 1, 2)$, Kaizhou District and Yunyang County can learn more knowledge in the process of implementing abatement technology than Wanzhou District. From Figure 9, it is clear that the pollution stocks decreased with the increasing of μ_i $(i = 1, 2)$. According to the definition of μ_i $(i = 1, 2)$, we can know only when more technologies are invested at every moment, then we can strengthen the ability to accumulate experience.

The effects of parameter μ_i $(i = 1, 2)$ on the noncooperative transboundary pollution problems in Wanzhou District, Kaizhou District and Yunyang County's abatement levels and pollution stocks have been shown in Figures 10–13.

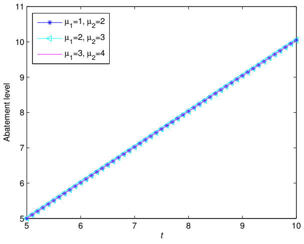

Figure 10. The effects of μ_i $(i = 1, 2)$ on the noncooperative transboundary pollution problems in Wanzhou District's abatement levels.

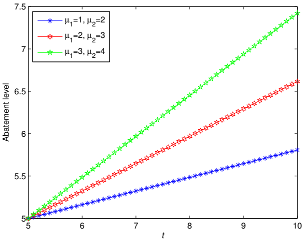

Figure 11. The effects of μ_i $(i = 1, 2)$ on the noncooperative transboundary pollution problems in Kaizhou District's abatement levels.

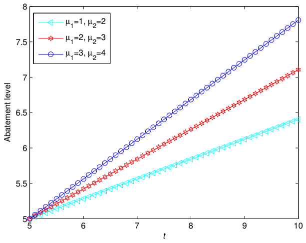

Figure 12. The effects of μ_i $(i = 1, 2)$ on the noncooperative transboundary pollution problems in Yunyang County's abatement levels.

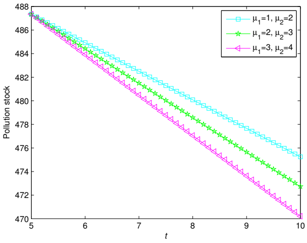

Figure 13. The effects of μ_i $(i = 1, 2)$ on pollution stock under noncooperative transboundary pollution problems.

Similarly, we can see that the effects of μ_i $(i = 1, 2)$ on the results of noncooperative are similar to those of cooperative implied in Figures 10–13. Furthermore, compared Figure 9 with Figure 13, it is obvious that with the same μ_i $(i = 1, 2)$, fewer pollution stocks of the cooperative transboundary pollution problems can be realized.

Conclusions

In this paper, we have discussed cooperative and noncooperative three transboundary pollution problems in Three Gorges Reservoir Area where the emission permits trading and abatement costs under learning by doing. The abatement cost depended on two key factors: the level of pollution abatement and the experience of using pollution abatement technology. We have established the optimal emission paths and the optimal abatement levels for the cooperative and noncooperative three transboundary pollution problems. Based on the actual economic data of Wanzhou District, Kaizhou District and Yunyang County, we obtained the abatement level and the pollution stock of cooperative and noncooperative three transboundary pollution problems. We also presented the effects of parameters μ_i $(i = 1, 2)$ for the abatement level and the pollution stock.

In future, we shall consider three transboundary pollution problems under learning by doing in the Beijing-Tianjin-Hebei Metropolitan Region.

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