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VORTICAL FLOW OF INCOMPRESSIBLE VISCOUS FLUID IN FINITE CYLINDER

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Abstract. The effective use of vortex energy in production of strong velocity fields by different devices is one of the modern areas of applications, developed during the last decade. In this paper the distribution of velocity field for viscous incompressible fluid in a pipe with a system of finite number of circular vortex lines, positioned on the inner surface of the finite cylinder is calculated. The approximation of the corresponding boundary value problem for the Navier–Stokes equations is based on the finite difference method. This procedure allows us to reduce the 2D problem decribed by the system of Navier- Stokes PDEs to the monotonous difference equations. Calculations are done using the computer program Matlab and the following regimes are calculated: a) the flow in a smooth pipe; b) the flow, poured inside a pipe through the circle; c) the flow, poured inside a pipe through the ring. The model is investigated for different values of parameters Re (Reynolds number), Γ (swirl number) and A (vortex intensity).

Key words: 2D problem, finite difference method, monotonous finite difference, Navier – Stokes equations, viscous fluid, monotonous finite difference schemes.

1 INTRODUCTION

For many new technological applications it is important to use vortex distributions in order to obtain large velocity fields [9]. Modern technologies require more efficient, compact and environmentally friendly technological equipment. This is very important in the field of the transformation of the alternating current into the heat energy [1]. During the last decade some inventors are trying to construct energetic devices based on the tornado principle. The practical aim of our investigation is to understand the process in the element of Hurricane Energy Transformer [8]. This element is the central point in the so called

RKA devices, which are used on the cars' roof for the substantial reduction of the air drag. In papers [2, 3] velocity fields of ideal incompressible fluid influenced by vortexes in a finite cylinder are investigated. The maximal value of the velocity was induced by the spiral vortexes in the middle of the cylinder.

In this paper we study the boundary – value problem for the system of three partial differential equations (PDEs) (i.e. the Navier – Stokes equations), which describes the stationary axially-symmetric flow of viscous incompressible liquid in a finite cylinder, or a pipe. The system of the finite number of circular vortex lines on the inner surface of the cylinder is considered. The discrete solution is obtained by the finite-difference method, and the discretization is based on the monotonous vector finite difference schemes (FDS) [4, 5, 6]. This procedure allows us to reduce the 2D problem decribed by the system of Navier – Stokes equations to the linear system of algebraic equations.

2 The Mathematical Model

In this paper we investigate the stationary axially–symmetric flow of incompressible liquid in a pipe. This process is described by the system of Navier – Stokes equations, which is given in the cylindrical coordinates (r, ϕ, z) [7]:

$$\begin{cases} M(V_z) = -\rho^{-1}\dfrac{\partial p}{\partial z} + \nu \Delta V_z + F_z \\ M(V_r) = -\rho^{-1}\dfrac{\partial p}{\partial r} + \nu(\Delta V_r - r^{-2}V_r) + r^{-1}V_\phi^2 + F_r \\ M(V_\phi) + r^{-1}V_r V\phi = \nu(\Delta V_z - r^{-2}V_\phi) \\ \dfrac{\partial(rV_r)}{\partial r} + \dfrac{\partial(rV_z)}{\partial z} = 0. \end{cases} \tag{2.1}$$

Here V_r, V_z, V_ϕ are the radial, axial and azimuthal components of the velocity vector V, depending on the coordinates r, z, Δ is the Laplace operator

$$\Delta g = r^{-1}\frac{\partial}{\partial r}\Big(r\frac{\partial g}{\partial r}\Big) + \frac{\partial^2 g}{\partial z^2},$$

F_r, F_z are components of the external force F,

$$M(g) = V_r\frac{\partial g}{\partial r} + V_z\frac{\partial g}{\partial z}$$

is the convective part of the equations, ρ, ν are the density and kinematic viscosity, $\eta = \rho\nu$ is the dynamic viscosity, p is pressure.

Eliminating pressure from the first two equations of the system (2.1) one obtains

$$M(\omega_\phi) - r^{-1}V_r\omega_\phi = \nu(\Delta\omega_\phi - r^{-2}\omega_\phi) + r^{-1}\frac{\partial V_\phi^2}{\partial z} + f_\phi, \tag{2.2}$$

where $\omega_\phi = \partial V_r/\partial z - \partial V_z/\partial r$ is the azimuthal component of $\operatorname{curl} V$ or the function of the vorticity, $f_\phi = \partial F_r/\partial z - \partial F_z/\partial r$ is the azimuthal component of curl F. The stream function ψ or the azimuthal component A_ϕ ($\psi = A_\phi r$) of velocity vector potential $V = rotA$ can be determined with formulas

$$V_r = -r^{-1}\frac{\partial\psi}{\partial z}, \quad V_z = r^{-1}\frac{\partial\psi}{\partial r}. \tag{2.3}$$

Then from the equation of continuity and from the vorticity function it follows, that

$$\omega_\phi = -r^{-1}\varDelta^*\psi, \tag{2.4}$$

where $\varDelta^*$ is the conjugated Laplace operator:

$$\varDelta^*\psi = r\partial(r^{-1}\partial\psi/\partial r)/\partial r + \partial^2\psi/\partial z^2.$$

From (2.1)–(2.4) we get the system of three PDEs for solving the circulation $W = rV_\phi$, transformed vorticity function $\omega = \omega_\phi/r$ and stream function ψ:

$$\begin{cases} r^{-1}J(\psi, W) = \nu\varDelta^* W \\ \varDelta^*\psi = -r^2\omega \\ r^{-1}J(\psi,\omega) = \nu\Big(\dfrac{\partial^2\omega}{\partial z^2} + r^{-3}\dfrac{\partial}{\partial r}\Big(r^3\dfrac{\partial\omega}{\partial r}\Big)\Big) + r^{-4}\dfrac{\partial W^2}{\partial z} + r^{-1}f_\phi, \end{cases} \tag{2.5}$$

where $J(\psi, g) = (\partial\psi/\partial r)(\partial g/\partial z) - (\partial\psi/\partial z)(\partial g/\partial r)$ is the Jacobian of the functions ψ and g, $g = \omega$ or W.

Let us consider the area of the pipe $\varOmega = \{(r, z) : 0 < r < a, 0 \le z \le L\}$, where a, L is the radius and length of the pipe. We write equations (2.5) in the dimensionless form by scaling all the lengths to $z_0 = r_0 = a$, the radial and axial velocities to U_0 (the uniform inlet velocity), the azimuthal velocity to $V_0 = a\varOmega_0$, the circulation to $W_0 = aV_0$, the vorticity function to $\omega_0 = U_0/a^2$, the stream function to $\psi_0 = U_0a^2$, the pressure to $p_0 = \rho U_0^2$, the external force to F_0, where $\varOmega_0$ is the angular velocity. The area of the pipe in the dimensionless form is $\varOmega = \{(r, z) : 0 < r < 1, 0 \le z \le l\}$, where $l = L/a$.

We have the following dimensionless form of the system (2.5)

$$\begin{cases} Re\, r^{-1}J(\psi, W) = \varDelta^* W, \\ \varDelta^*\psi = -r^2\omega, \\ Re\, r^{-1}J(\psi,\omega) = \dfrac{\partial^2\omega}{\partial z^2} + r^{-3}\dfrac{\partial}{\partial r}\Big(r^3\dfrac{\partial\omega}{\partial r}\Big) + \dfrac{Re\,\varGamma^2}{r^4}\dfrac{\partial W^2}{\partial z} + F_V r^{-1}f_\phi, \end{cases} \tag{2.6}$$

where $Re = (U_0a)/\nu$ is the Reynolds number, $\varGamma = V_0/U_0$ is the swirl number, $F_V = F_0a^2/(U_0\nu)$ is the parameter of the source.

We have the following dimensionless form of the boundary conditions:

1) on the inlet of the pipe ($z = 0$) the axial velocity U_0 or/and the swirl velocity W_0 are given,

2) the conditions of symmetry are given on the axis ($r = 0$):

$$\psi = \partial\psi/\partial r = \partial\omega/\partial r = W = 0,$$

3) the outflow conditions are specified at the outlet ($z = l$):

$$\partial\psi/\partial z = \partial\omega/\partial z = \partial W/\partial z = 0,$$

4) the non-slipping conditions are defined on the wall ($r = 1$):

$$\psi = 0.5, \quad \partial\psi/\partial r = W = 0,$$

5) special conditions are given for the vorticity function $\omega = \omega_W + G(z)$, where ω_W is the value of the vorticity on the wall (the modified Wood conditions). They characterize the non-slip of the liquid on the wall, $G(z) = A\sin^{100}(\pi k(z - l_1))$ (in (2.6) we take $f_\phi = 0$) is the function, which generates circular vortex lines in the wall segment $z \in [l_1, l_2]$, $l_1 > 0, l_2 < l$ and outside of this segment $G(z) = 0$). The centers of vortexes are defined by $z_i = l_1 + (i - 0.5) * (l_2 - l_1)/M, i = \overline{1, M}$, $k = M/(l_2 - l_1)$, here M, A are the number and the intensity of vortexes.

If $f_\phi \neq 0$, then we have in the neighbourhood of the centre of vortexes the following components of external force: $F_r = Az/F_V,\ F_z = 4Ar/F_V$.

In this paper, we consider three different uniform flow conditions on the inlet $z = 0$, where the liquid is poured inside the pipe.

1. If we have the Poiseuille flow (i.e. $V_r = 0$, $V_z = V_z(r)$, $\partial p/\partial z = const$), then from (2.6) it follows that

$$\psi := \psi(r) = -\frac{C_1}{4} r^2(\ln r - 0.5) + \frac{C_2}{8} r^4 + \frac{C_3}{2} r^2 + C_4, \tag{2.7}$$

where $C_k, k = 1, 2, 3, 4$ are arbitrary constants. The expression of stream function $\psi(r) = r^2 - 0.5r^4$, $\omega = 4$ follows from the boundary conditions

$$\psi(0) = \psi'(0) = \psi'(1) = 0, \quad \psi(1) = 0.5.$$

2. The flow through the ring $0 < r_* < r \leq 1$, where the entrance part of the pipe $0 < r \leq r_*$ is closed. Then from (2.6) and from conditions

$$\psi(r_*) = \psi'(r_*) = \psi'(1) = 0, \quad \psi(1) = 0.5,$$

it follows, that

$$\psi(r) = -\frac{r^2(\ln r - 0.5)}{g} + \frac{r^4 \ln r_* - 2r^2 \ln r_* + r_*^2(g - \ln r_*)}{2g(r_*^2 - 1)},$$

$$\omega(r) = \frac{1}{g}\Big(\frac{1}{r^2} - \frac{2\ln r_*}{(r_*^2 - 1)}\Big),$$

where $g = \ln r_*(r_*^2 + 1) + 1 - r_*^2 > 0$. Note, that if $r_* \to 0$, then we obtain the Poiseuille flow.

3. The flow through the circle $0 < r < r_* \leq 1$ and the input stream with the azimuthal velocity $V_\phi(r_*) = \Omega_1 r_*$, $V_\phi(1) = \Omega_2$, where Ω_1, Ω_2 are the angular velocity for the barrier and the wall of pipe. From the equation of motion (2.1) we have the azimuthal velocity in the following form

$$V_\phi(r) = (1 - r_*^2)(r(\Omega_2 - \Omega_1 r_*^2) - r_*^2(\Omega_2 - \Omega_1)/r). \tag{2.8}$$

3 The Finite-Difference Scheme

For the approximation of the convective and diffusive terms special monotonous approximations are used [6]. The vorticity equation ($\Gamma = 0$) in the uniform grid (r_i, z_j) is replaced by monotonous difference equations of the second order approximation:

$$B^z_{i,j}(\omega_{i,j+1}-\omega_{i,j})-A^z_{i,j}(\omega_{i,j}-\omega_{i,j-1})+B^r_{i,j}(\omega_{i+1,j}-\omega_{i,j})-A^r_{i,j}(\omega_{i,j}-\omega_{i-1,j}) = 0, \tag{3.1}$$

where

$$B^z_{i,j} = h_2^{-2}S(Re\,V_{z,i,j+0.5}h_2), \quad A^z_{i,j} = h_2^{-2}S(-Re\,V_{z,i,j-0.5}h_2),$$

$$B^r_{i,j} = h_1^{-2}\frac{r^3_{i+0.5}}{r^3_i}S(Re\,V_{r,i+0.5,j}h_1), \quad A^r_{i,j} = h_1^{-2}\frac{r^3_{i-0.5}}{r^3_i}S(-Re\,V_{r,i-0.5,j}h_1),$$

$$r_{i\pm0.5} = r_i \pm 0.5h_1, \quad i = 1,\dots,n_r, \ \ j = 2,\dots,n_z,$$

$$S(x) = x/(e^x - 1) > 0, \quad \omega_{i,j} \approx \omega(r_i, z_j),$$

$$r_i = (i-0.5)h_1, \ \ z_j = (j-1)h_2, \quad h_1 = 1/n_r, \ \ h_2 = l/n_z.$$

In a similar way, the vector–monotonous finite-difference equations can be obtained for functions ω, W in the case $\Gamma \neq 0$ [6].

If $i \neq 1$, the approximation of the second equation in (2.6) can be obtained in the standard form [6]. If $i = 1, r_1 = 0.5h_1$, then taking into account that $\psi_{0.5,j} = 0$, $\psi_{0,j} = \psi_{1,j}$, it follows from the PDEs that for $r = 0$ we have

$$V_z = r^{-1}\frac{\partial\psi}{\partial r} = \frac{\partial^2\psi}{\partial r^2} \approx \frac{\psi_{1,j} - 2\psi_{0.5,j} + \psi_{0,j}}{(0.5h_1)^2}$$

and the approximation of the expression $r\dfrac{\partial}{\partial r}\Big(\dfrac{1}{r}\dfrac{\partial\psi}{\partial r}\Big)$ is done in the form

$$0.5h_1^{-2}(\psi_{2,j} - 9\psi_{1,j}).$$

The approximation of the first equation in (2.6) for $i \neq 1$ is done in the form (3.1), where $\omega = W$,

$$B^r_{i,j} = \frac{1}{h_1^2}\frac{r_i}{r_{i+0.5}}\,S(ReV_{r,i+0.5,j}h_1), \quad A^r_{i,j} = \frac{1}{h_1^2}\frac{r_i}{r_{i-0.5}}S(-ReV_{r,i-0.5,j}h_1).$$

In the case of $i = 1$, taking into account $W_{0.5,j} = 0$, $W_{0,j} = W_{1,j}$ it follows from the first equation of (2.6), that for $r = 0$ we get the approximation

$$A_1 = r\frac{\partial}{\partial r}\Big(\frac{1}{r}\frac{\partial W}{\partial r}\Big) = \frac{r_1}{h_1}\Big(\frac{1}{r_1 + 0.5h_1}\frac{\partial W_{1.5,j}}{\partial r} - \frac{\partial^2 W_{0.5,j}}{\partial r^2}\Big) + O(h_1^2)$$

$$= \frac{r_1}{h_1}\Big(\frac{W_{2,j} - W_{1,j}}{h_1^2} - \frac{8}{h_1}W_{1,j}\Big) + O(h_1^2).$$

For $i = 1$, the approximation of the second order of accuracy can be presented in the previous form, where

$$A^r_{1,j} = \frac{8r_1}{h_1}, \quad W_{0,j} = 0.$$

For the boundary conditions on the wall ($r = 1$) of the pipe the following Taylor series are used :

$$\begin{cases} \psi_{nr} = \psi_W - 0.5h_1 \dfrac{\partial \psi_W}{\partial r} + \dfrac{h_1^2}{8}\dfrac{\partial^2 \psi_W}{\partial r^2} - \dfrac{h_1^3}{48}\dfrac{\partial^3 \psi_W}{\partial r^3} + O(h_1^4), \\ \omega_{nr} = \omega_W - 0.5h_1 \dfrac{\partial \omega_W}{\partial r} + O(h_1^2), \end{cases} \tag{3.2}$$

where ψ_W, ω_W are the values of the appropriate functions defined on the wall. The boundary conditions

$$\frac{\partial \psi_W}{\partial r} = 0, \quad \frac{\partial^2 \psi_W}{\partial r^2} = -r^2 \omega_W, \quad \frac{\partial^3 \psi_W}{\partial r^3} = -r^2 \frac{\partial \omega_W}{\partial r} - 3r\omega_W$$

follow from the non-slip conditions and from the second equation (2.6).

Using the expression

$$\frac{\partial \omega_W}{\partial r} = 2\frac{\omega_W - \omega_{nr}}{h_1} + O(h_1)$$

we write the modified Wood [6] conditions in the form

$$\omega_W = -\frac{1}{1 - 0.75h_1}\Big(\frac{12}{h_1^2}(\psi_{nr} - \psi_W) + 0.5\omega_{nr}\Big). \tag{3.3}$$

The modified Wood conditions

$$\omega_{i,1} = -\frac{3}{h_2^2 r_i^2}(\psi_{i,2} - \psi_{i,1}) - 0.5\omega_{i,2}. \tag{3.4}$$

for the flow in a ring pipe on the wall $z = 0, 0 < r \le r_*$ follow from Taylor's series.

4 Calculation of Pressure on the Axis of the Pipe

From the first dimensionless equation (2.1) we obtain ($F_z = 0$)

$$\frac{\partial p}{\partial z} = \Delta V_z - 0.5Re\,\frac{\partial V_z^2}{\partial z}.$$

Using condition (2.6) and the Lopital rule we get that

$$r\frac{\partial V_z}{\partial r} = -r^2\omega - \frac{\partial^2 \psi}{\partial z^2}, \quad \frac{1}{r}\frac{\partial}{\partial r}\Big(r\frac{\partial V_z}{\partial r}\Big) = -2\omega = \frac{\partial^4 \psi}{\partial r^2 \partial z^2},$$
$$\frac{\partial^2 V_z}{\partial z^2} = \frac{\partial^4 \psi}{\partial r^2 \partial z^2}, \quad \Delta V_z = -2\omega, \quad \frac{\partial V_z^2}{\partial z} = \frac{\partial}{\partial z}\Big(\frac{\partial^2 \psi}{\partial r^2}\Big)^2.$$

Therefore, if $r = 0$, then

$$\frac{\partial p}{\partial z} = -2\omega - 0.5Re\,\frac{\partial}{\partial z}\Big(\frac{\partial^2\psi}{\partial r^2}\Big)^2. \tag{4.1}$$

From the Poiseuille flow ($\omega = 4, \partial\psi/\partial z = 0$) it follows, that

$$\frac{\partial p}{\partial z} = -8, \quad \nabla p = p|_{z=0} - p_{z=l} = 8l.$$

We consider the approximation of the integral equality

$$\int_0^l \frac{\partial p}{\partial z}\,dz = -\nabla p$$

by using the finite-difference and quadrature formulae. For $z = z_{j-0.5}$ we get the approximation

$$\frac{\partial p}{\partial z} \approx -2\omega_{0.5,j-0.5} - \frac{Re}{2h_2}\Big[\Big(\frac{\partial^2\psi}{\partial r^2}\Big)^2\Big|_{0.5,j} - \Big(\frac{\partial^2\psi}{\partial r^2}\Big)^2\Big|_{0.5,j-1}\Big], \ \ j = 1,\dots,n_z.$$

The derivative $\Big(\dfrac{\partial^2\psi}{\partial r^2}\Big)^2$ can be calculated by means of the Taylor series:

$$\psi_{1,j} = \psi_{0.5,j} + \Big(\frac{h_1}{2}\frac{\partial\psi}{\partial r} + \frac{h_1^2}{8}\frac{\partial^2\psi}{\partial r^2} + \frac{h_1^3}{48}\frac{\partial^3\psi}{\partial r^3} + \frac{h_1^4}{384}\frac{\partial^4\psi}{\partial r^4}\Big)\Big|_{0.5,j} + O(h_1^5),$$

$$\psi_{2,j} = \psi_{0.5,j} + \Big(\frac{3h_1}{2}\frac{\partial\psi}{\partial r} + \frac{9h_1^2}{8}\frac{\partial^2\psi}{\partial r^2} + \frac{27h_1^3}{48}\frac{\partial^3\psi}{\partial r^3} + \frac{81h_1^4}{384}\frac{\partial^4\psi}{\partial r^4}\Big)\Big|_{0.5,j} + O(h_1^5).$$

Taking into account, that

$$\psi_{0.5,j} = \frac{\partial\psi}{\partial r}\Big|_{0.5,j} = 0$$

and multiplying the first expression by 81 and subtracting it from the second expression we get

$$\frac{\partial^2\psi}{\partial r^2}\Big|_{0.5,j} = \frac{81\psi_{1,j} - \psi_{2,j}}{2.25h_1^2} + O(h_1^2). \tag{4.2}$$

We extrapolate the value $\omega_{0.5,j-0.5} = 0.5(\omega_{0.5,j} + \omega_{0.5,j-1})$ by using the Taylor series:

$$\omega_{1,j} = \omega_{0.5,j} + \frac{h_1}{2}\Big(\frac{\partial\omega}{\partial r}\Big)_{0.5,j} + \frac{h_1^2}{8}\Big(\frac{\partial^2\omega}{\partial r^2}\Big)_{0.5,j} + \frac{h_1^3}{48}\Big(\frac{\partial^3\omega}{\partial r^3}\Big)_{0.5,j} + O(h_1^4),$$

$$\omega_{2,j} = \omega_{0.5,j} + \frac{3h_1}{2}\Big(\frac{\partial\omega}{\partial r}\Big)_{0.5,j} + \frac{9h_1^2}{8}\Big(\frac{\partial^2\omega}{\partial r^2}\Big)_{0.5,j} + \frac{27h_1^3}{48}\Big(\frac{\partial^3\omega}{\partial r^3}\Big)_{0.5,j} + O(h_1^4).$$

Taking into account, that $(\partial\omega/\partial r)_{0.5,j} = (\partial^3\omega/\partial r^3)_{0.5,j} = 0$ and multiplying the first expression by 9 and subtracting it from the second one we obtain

$$\omega_{0.5,j} = \frac{9}{8}\omega_{1,j} - \frac{1}{8}\omega_{2,j} + O(h_1^4). \tag{4.3}$$

After omitting the members of $O(h_1^2)$, $O(h_1^4)$ in (4.1, 4.3), we find the approximation formulae of the second order for the pressure gradient on the axis

$$\left(\frac{\partial p}{\partial z}\right)_{0.5,j-0.5} = -\frac{9\omega_{1,j} - \omega_{2,j} + 9\omega_{1,j-1} - \omega_{2,j-1}}{8} - Re\frac{81\psi_{1,j} - \psi_{2,j} - 81\psi_{1,j-1} + \psi_{2,j-1}}{4.5h_2h_1^2}. \tag{4.4}$$

Then the integral can be calculated with the help of the summation formula

$$-\nabla p = h_2 \sum_{j=1}^{n_z} \left(\frac{\partial p}{\partial z}\right)_{0.5,j-0.5}. \tag{4.5}$$

5 Results of Numerical Experiments

All computations and graphic visualization of results were done by means of mathematical system MATLAB-7. We use the following values of parameters:

$$Re \in [1, 500],\ \ \Gamma = 0; 2,\ \ F_V = 1,\ \ r_* = 0.475,\ \ M = 4,\ \ A = 0; \pm 2000,$$
$$z_1 = 0.75,\ \ z_2 = 1.25,\ \ z_3 = 1.75,\ \ z_4 = 2.25,\ \ l_1 = 0.5,\ \ l_2 = 2.5,$$
$$l = 4,\ \ h_1 = 0.01,\ \ h_2 = 0.05,\ \ n_z = 80,\ \ n_r = 20.$$

The finite-difference equations are solved by the method of sub-relaxation with the relaxation parameter in the interval (0.3, 0.8). Depending on different flow conditions specified on the inlet of the pipe, we have obtained numerical results for three experimental situations.

1. **Flow in a smooth pipe.**

Depending on the intensity of vorticity A different values of the pressure difference $-\nabla p$ (see Table 1) and the distribution of the pressure on the axis of the cylinder were obtained (see Fig. 1 for $A = 0$ and Fig. 2 for $A = -2000$).

Table 1. Dependence of the pressure drop $(-\nabla p)$ on the values of A.

A	0	500	1000	2000	-500	-1000	-2000
$-\nabla p$	32	22	18	12	38	46	53

In the figures the coordinate Oz are given on the horizontal axis and numerical values of pressure are presented on the vertical axis.

In the case of $A > 0$, we get that when vortices intensity A on the wall is increasing the pressure drop $(-\nabla p)$ decreases and the stream velocity increases, but for decreasing of vortices intensity ($A < 0$ means that the vortices rotate in an opposite direction) the pressure drop increases and the stream velocity decreases.

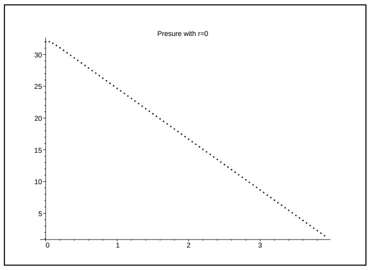

Figure 1. The distribution of pressure on an axis in smooth pipe $A = 0$.

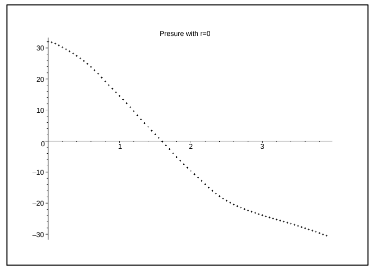

Figure 2. The distribution of pressure on an axis in smooth pipe $A = -2000$.

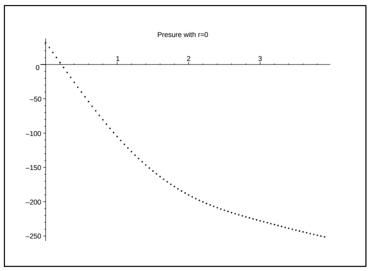

Figure 3. The distribution of pressure on an axis with the reduced input, $Re = 10$.

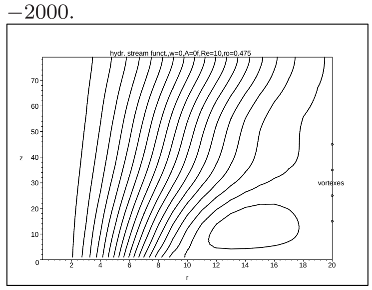

Figure 4. Streamlines of velocity in a pipe with the reduced input, $Re = 10, A = 0$.

2. The flow through the circle $0 < r < r_* \leq 1$.

If $\varGamma = \varOmega_1 = A = 0$, then depending on values of parameter Re various values of pressure drop (see Table 2) are received. Distributions of the pressure and stream functions are given accordingly in Fig. 3 and Fig. 4.

Table 2. Dependence of the pressure drop ($-\nabla p$) on the values of Re.

Re	0	1	10	20
$-\nabla p$	69	82	283	440

It follows from numerical results that for increased values of Re the vortices zone on the inlet is increased too.

If $A = 2000$, then the stream in a vortices zone varies essentially (see Fig. 5). Having values of parameter $\varOmega_1 = 1, Re = 10, A = 0$ the drop of the pressure is equal to 270, and for values $A = 2000$ and $A = -2000$ we get the following values of the pressure drop $\nabla = -267$ and $\nabla = -272$.

3. The flow through a ring $r_* < r \leq 1$.

In Fig. 6 the distribution of a stream function is presented for

$$\varGamma = A = \varOmega_1 = \varOmega_2 = 0, \quad Re = 100.$$

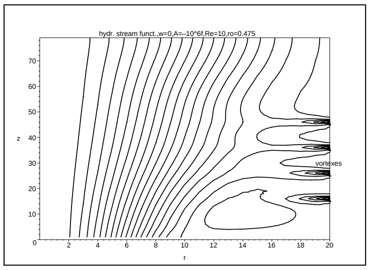

Figure 5. Streamlines of velocity in a pipe with the reduced input, $Re = 10, A = 2000$.

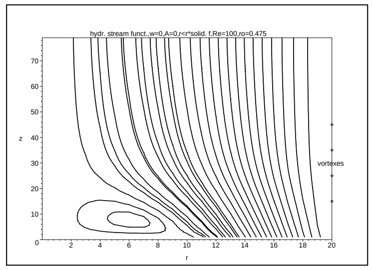

Figure 6. Streamlines of velocity in a pipe with the ring input, $Re = 100, A = 0$.

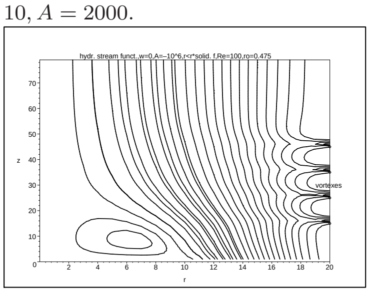

Figure 7. Streamlines of velocity in a pipe with a ring input, $Re = 100, A = 2000$.

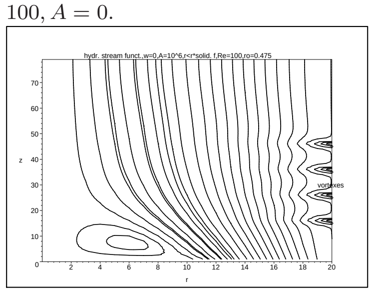

Figure 8. Streamlines of velocity in a pipe with the ring input, $Re = 100, A = -2000$.

In Fig. 7 and Fig. 8 the stream lines are given for $A = 2000$ and $A = -2000$. If a rotation takes place (i.e. we take the following parameters $\Gamma = 2$, $\Omega_1 = 0$, $\Omega_2 = 1$, or $\Omega_1 = 1$, $\Omega_2 = 0$ and $A = \pm 2000$), then the distributions of stream functions are very similar and presented in Fig. 7 and Fig. 8.

It is necessary to note, that the streamlines do not depend on changes of a rotation direction. In Fig. 9 and Fig. 10 we can see the distribution of the circulation and the vorticity function for the following values of parameters $A = 0$, $\Omega_1 = 0$, $\Omega_2 = 1$, $Re = 100$.

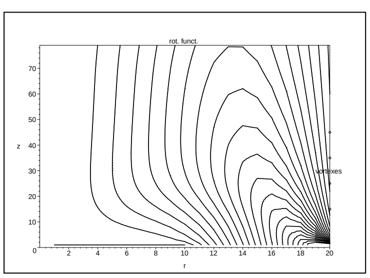

Figure 9. Distribution of the circulation with a ring input, $A = 0, Re = 100, \Omega_1 = 0, \Omega_2 = 1$.

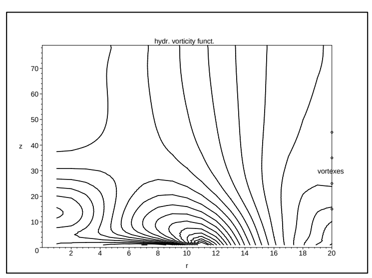

Figure 10. Distribution of the vorticity function with a ring input, $A = 0, Re = 100, \Omega_1 = 0, \Omega_2 = 1$.

6 Conclusions

1. Acceleration of a liquid stream on the wall of the pipe is achieved by the creation of the vertical stream in the clockwise direction.
2. The character of a stream of liquid in a pipe depends on the values of the Reynolds number Re, the swirl number Γ and the vortex intensity on the wall A.
3. The character of a stream of liquid in a pipe depends on the liquid input mode pouring inside of the pipe.
4. The considered technique is applicable in specific targets and processes of formation of vortexes, for example, a stream of a liquid or gas in pipelines, automobile mufflers, etc.

References

[1] A. Buikis and H. Kalis. The vortex formation in a horizontal finite cylinder by alternating electric current. *Math. Model. Anal.*, **10**(1):9–18, 2005.

[2] A. Buikis, H. Kalis and J. Schatz. Velocity field in the ideal fluid, induced by vortex curves in a finite cylinder and cone. *WSEAS Transactions on Environment and Development*, **1**(1):26–32, 2005.

[3] A. Buikis, H. Kalis and J. Schatz. Numerical calculation of flow field caused by vortices chain. In *In Proc. of the 5th WSEAS International Conference on System Science and Simulation in Engineering*, pp. 427–432. Tenerife, Canary Islands, Spain, December 16-18, 2006.

[4] R. Čiegis, O. Iliev and Z. Lakdawala. On parallel numerical algorithms for simulating industrial filtration problems. *Computational Methods in Applied Mathematics*, **7**(2):118–134, 2007.

[5] R. Čiegis, O. Iliev, V. Starikovičius and K. Steiner. Numerical algorithms for solving problems of multiphase flows in porous media. *Math. Model. and Anal.*, **11**(2):133–148, 2006.

[6] H. Kalis. Special computational methods for solution of MHD problems. *MHD*, **30**:144–155, 1994.

[7] J.E. Martin and E.Meilung. Nonlinear axisymmetric and three-dimensional vorticity dynamics in a swirling jet mode. *L. Phys. Fluids*, **8**, 1996.

[8] J. Schatz. Mini-Tornado mit Wirbeleffekten. *Neue Universale Energielösungen*, pp. 247–290, 2003.

[9] S.E. Widnall. The structure and dynamics of vortex filaments. *Annual Review of Fluid Mechanics*, **7**:141–166, 1975.