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Application of the Caputo-Fabrizio Fractional Derivative without Singular Kernel to Korteweg-de Vries-Burgers Equation*

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Abstract. In order to bring a broader outlook on some unusual irregularities observed in wave motions and liquids' movements, we explore the possibility of extending the analysis of Korteweg–de Vries–Burgers equation with two perturbation's levels to the concepts of fractional differentiation with no singularity. We make use of the newly developed Caputo-Fabrizio fractional derivative with no singular kernel to establish the model. For existence and uniqueness of the continuous solution to the model, conditions on the perturbation parameters ν, μ and the derivative order α are provided. Numerical approximations are performed for some values of the perturbation parameters. This shows similar behaviors of the solution for close values of the fractional order α.

Keywords: Caputo-Fabrizio fractional derivative, non-linear Korteweg-de Vries-Burgers equation, existence and uniqueness, perturbation, numerical solutions.

AMS Subject Classification: 26A33; 34A12; 35D05.

1 Introduction

Most of real and complex world problems appearing in many branches of engineering, natural sciences and technology are mathematically described via nonlinear partial differential equations, often very hard to handle in terms of providing their exact behavior or expressing their exact solutions. However, today it is widely known that the Newtonian concept of derivative can no longer satisfy all the complexity of real life problems and many of those problems remain unsolved and open [1, 2, 9, 10, 12, 15, 19, 20]. Among those complex issues we count phenomena related to the study of chaos in waves motion and solitary waves. Hence, the Korteweg–de Vries–Burgers (KDVB) equation is used to describe and analyze some physical contexts related to liquids and waves

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dynamics. For example, it has be used in [14] to investigate the propagation of waves in an elastic tube filled with a viscous fluid, in [4] to analyze the propagation of undular bores in shallow water and in [11, 13] to study the behavior of weakly nonlinear plasma waves with certain dissipative effects. The authors in [8] performed a qualitative analysis to a two-dimensional autonomous KDVB equation and pointed out that under certain conditions, the Korteweg-de Vries-Burgers equation has neither nontrivial bell-profile solitary waves, nor periodic waves.

However, due to increasing irregularities and nonlinearities observed in liquids and waves motion in general [5, 8, 11, 13, 16, 21], it is necessity to establish broader outlooks on them. This is why there is a growing volition to extend classical models to new models with time fractional derivative and investigate them with various and different techniques. In this paper, we investigate the possibility of extending the analysis of the KDVB equation to the concept of time fractional differentiation. The derivative in use here is not any one, but the newly introduced time fractional order derivative without singular kernel [7,17].

2 History of fractional order derivatives with no singular kernel

One of the greatest attempts to enhance nonlinear mathematical models was to introduce the concept of derivative with fractional order. The literature comprises many definitions of fractional derivative ranging from local to non-local type [6, 10, 19, 20]. The most popular remain the Riemann–Liouville and the Caputo derivatives respectively defined as

$$D_x^\alpha\left(u(x)\right)=\frac{1}{\Gamma\left(n-\alpha\right)}\left(\frac{d}{dx}\right)^n\int_0^x\left(x-t\right)^{n-\alpha-1}u\left(t\right)dt,\quad n-1<\alpha\leq n$$

and

$$D_x^\alpha\left(u(x)\right)=\frac{1}{\Gamma\left(n-\alpha\right)}\int_0^x\left(x-t\right)^{n-\alpha-1}\left(\frac{d}{dt}\right)^n u\left(t\right)dt,\quad n-1<\alpha\leq n.$$

However, they appear to be particularly suitable to describe physiscal phenomena, related to fatigue, damage and electromagnetic hysteresis, but are incapable of properly describing some behavior observed in materials with huge heterogeneities and structures with different scales. Hence, Caputo and Fabrizio [7] developed and proposed a new version of derivative with fraction order that is defined as follows:

Definition 1 [**Caputo-Fabrizio derivative with fractional order (CFFD)**]. Let u be a function in $H^1(a;b)$; $b>a$; $\alpha\in[0;1]$ then, the new Caputo derivative of fractional order α is defined as:

$${}^{cf}D_t^\alpha u(t)=\frac{M(\alpha)}{(1-\alpha)}\int_0^t\dot{u}\left(\tau\right)\exp\left(-\frac{\alpha(t-\tau)}{1-\alpha}\right)d\tau,$$

where $M(\alpha)$ is a normalization function such that $M(0) = M(1) = 1$. But, for the function that does not belong to $H^1(a;b)$, we defined its Caputo-Fabrizio fractional as

$$^{cf}D_t^\alpha u(t) = \frac{\alpha M(\alpha)}{(1-\alpha)} \int_0^t (u(t) - u(\tau)) \exp\left(-\frac{\alpha(t-\tau)}{1-\alpha}\right) d\tau.$$

The definition of the CFFD was improved by Losada and Nieto [17] to become

$$^{cf}D_t^\alpha u(t) = \frac{(2-\alpha)M(\alpha)}{2(1-\alpha)} \int_0^t \dot{u}(\tau) \exp\left(-\frac{\alpha(t-\tau)}{1-\alpha}\right) d\tau. \tag{2.1}$$

Unlike the classical version of Caputo fractional order derivative [6, 18], the new CFFD has no singular kernel due to the substitution of the kernel $\frac{1}{(t-\tau)^\alpha}$ appearing in the classical definition. Moreover the CFFD satisfies the following relations for any suitable function u:

$$\lim_{\alpha\to 1} {}^{cf}D_t^\alpha u(t) = \dot{u}(t), \quad \lim_{\alpha\to 0} {}^{cf}D_t^\alpha u(t) = u(t) - u(a), \tag{2.2}$$

where a is the starting point of the integro-differentiation. The fractional integral (anti-derivative) associated to the CFFD was proposed as well by Losada and Nieto and proved to be:

$$^{cf}I_t^\alpha u(t) = \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} u(t) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t u(\tau)\, d\tau, \tag{2.3}$$

$\alpha \in [0,1]$ $t \geq 0$. This is anti-derivative is seen as kind of an average between function u and its integral of order one. The Laplace transform of the Caputo-Fabrizio fractional derivative (CFFD) is given by

$$\mathcal{L}\left(D_t^\alpha u(t), s\right) = \frac{s\tilde{u}(x,s) - u_0(x)}{s + \alpha(1-s)},$$

where $\tilde{u}(x,s)$ is the Laplace transform $\mathcal{L}(u(x,t),s)$ of $u(x,t)$.

Moreover, in reaction to the newly introduced Caputo-Fabrizio fractional derivative without singular kernel and being aware of the conflicting situations that exist between the classical Riemann-Liouville and Caputo derivatives, Doungmo Goufo and Atangana recently developed a new definition of fractional derivative generated by modification of the classical Riemann-Liouville definition and called the new Riemann-Liouville fractional derivative without singular kernel (NRLFD) expressed for $\alpha \in [0,1]$ as

$$_a\mathfrak{D}_t^\alpha u(t) = \frac{M(\alpha)}{1-\alpha} \frac{d}{dt} \int_a^t u(\tau) \exp\left(-\frac{\alpha}{1-\alpha}(t-\tau)\right) d\tau.$$

Again, contrary to the classical version of Riemann-Liouville fractional derivative, the NRLFD is without any singularity at $t = \tau$ and satisfies

$$\lim_{\alpha\to 1} {}_a\mathfrak{D}_t^\alpha u(t) = \dot{u}(t), \quad \lim_{\alpha\to 0} {}_a\mathfrak{D}_t^\alpha u(t) = u(t).$$

The Laplace transform of the NRLFD was proved to be

$$\mathcal{L}\left(\mathfrak{D}_t^{-\alpha}u(t), s\right) = \frac{sM(\alpha)}{s+\alpha(1-s)}\mathcal{L}(u(t),\ s).$$

The analyzis in this paper is performed making use of the new Caputo-Fabrizio fractional derivative without any singularity.

3 Existence and uniqueness

We aim to show in this section the existence and uniqueness of the Korteweg–de Vries–Burgers (KDVB) equation with two perturbation's levels using the new Caputo-Fabrizio fractional derivative with no singular kernel. We make use of the improved version (2.1) because of its anti-derivative (2.3) that is explicitly and fully given. Hence, the equation reads as

$$^{cf}D_t^{\alpha}u(x,t) = \nu u_{xx} - 2uu_x - \mu u_{xxx}, \tag{3.1}$$

where ν and μ are the perturbation parameters, $^{cf}D_t^{\alpha}$ is the CFFD given in (2.1) with initial condition

$$u(x,0) = h(x). \tag{3.2}$$

To proceed with existence results for the model (3.1)–(3.2), we exploit the expression of integral (2.3). Then,

$$u(x,t) - u(x,0) = ^{cf} I_t^{\alpha}\left(\nu u_{xx} - 2uu_x - \mu u_{xxx}\right).$$

Equivalently,

$$u(x,t) - u(x,0) = \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\left(\nu u_{xx} - 2uu_x - \mu u_{xxx}\right) \tag{3.3}$$

$$+\frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \left(\nu u_{xx} - 2uu_x - \mu u_{xxx}\right)d\tau.$$

Let us now put

$$\mathcal{N}(x,t,u,\nu,\mu) = \nu u_{xx} - 2uu_x - \mu u_{xxx}.$$

We have to find a positive real number L such that

$$\|\mathcal{N}(x,t,u,\nu,\mu) - \mathcal{N}(x,t,v,\nu,\mu)\| \leq L\|u-v\|.$$

In fact

$$\begin{aligned}\mathcal{N}(x,t,u,\nu,\mu) &- \mathcal{N}(x,t,v,\nu,\mu)\\ &= (\nu u_{xx} - 2uu_x - \mu u_{xxx}) - (\nu v_{xx} - 2vv_x - \mu v_{xxx})\\ &= \nu(u_{xx} - v_{xx}) + 2(vv_x - uu_x) + \mu(v_{xxx} - u_{xxx}).\end{aligned}$$

Exploiting the norm's properties leads to

$$\begin{aligned}
&\|\mathcal{N}(x,t,u,\nu,\mu)-\mathcal{N}(x,t,v,\nu,\mu)\| \\
&\qquad = \|\nu(u_{xx}-v_{xx})+2(vv_x-uu_x)+\mu(v_{xxx}-u_{xxx})\| \\
&\qquad \le \nu\|u_{xx}-v_{xx}\|+2\|vv_x-uu_x\|+\mu\|v_{xxx}-u_{xxx}\| \\
&\qquad \le \nu\|\partial_{xx}(u-v)\|+\|\partial_x(v^2-u^2)\|+\mu\|\partial_{xxx}(v-u)\|.
\end{aligned}$$

Because of assumption that u and v are bounded, there is a positive constant $c>0$ such that $\|u\|\le c$ and $\|v\|\le c$. Then, their first order derivative function ∂_x satisfies the Lipschitz condition and there is a number $L_1\ge 0$ such that

$$\begin{aligned}
&\|\mathcal{N}\left(x,t,u,\nu,\mu\right)-\mathcal{N}\left(x,t,v,\nu,\mu\right)\| \\
&\qquad \le \nu L_1^2\|u-v\|+L_1\|v^2-u^2\|+\mu L_1^3\|u-v\| \\
&\qquad \le \nu L_1^2\|u-v\|+L_1\|u+v\|\cdot\|u-v\|+\mu L_1^3\|u-v\| \\
&\qquad \le \left[\nu L_1^2+2cL_1+\mu L_1^3\right]\|u-v\|,
\end{aligned}$$

where we have used the bounded condition (3.2). Hence,

$$\|\mathcal{N}(x,t,u,\nu,\mu)-\mathcal{N}(x,t,v,\nu,\mu)\|\le L\|u-v\|$$

with $L=\nu L_1^2+2cL_1+\mu L_1^3$. This shows the Lipschitz condition for $\mathcal{N}$. Now we can state the following theorem.

Proposition 1. *Under the condition that* $\frac{2L(1-\alpha)}{(2-\alpha)M(\alpha)}+\frac{2Lt\alpha}{(2-\alpha)M(\alpha)}<1$, *then the non-linear time fractional Korteweg–de Vries–Burgers model with two perturbation's levels and no singular kernel*

$$\begin{cases} {}^{cf}D_t^{\alpha}u(x,t)=\nu u_{xx}-2uu_x-\mu u_{xxx} \\ u(x,0)=h(x) \end{cases} \tag{3.4}$$

admits a unique solution that is continuous.

Proof. To prove it, we consider (3.3):

$$\begin{aligned}
&u(x,t)-u(x,0) \\
&\quad =\frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\mathcal{N}(x,t,u,\nu,\mu)+\frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t\mathcal{N}(x,\tau,u,\nu,\mu)d\tau,
\end{aligned}$$

that suggests the following recurrence formula

$$\begin{aligned}
u_0(x,t)&=u(x,0),\\
u_n(x,t)&=\frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\mathcal{N}(x,t,u_{n-1},\nu,\mu)\\
&\quad+\frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t\mathcal{N}(x,\tau,u_{n-1},\nu,\mu)d\tau.
\end{aligned}$$

Let

$$\bar{u}(x,t) = \lim_{n\to\infty} u_n(x,t). \tag{3.5}$$

We aim to show that $\bar{u}(x,t) = u(x,t)$ is a solution that is continuous. Let us set

$$G_n(x,t) = u_n(x,t) - u_{n-1}(x,t).$$

It is obvious that

$$u_n(x,t) = \sum_{m=0}^{n} G_m(x,t).$$

Furthermore, in a more detailed way we have

$$G_n(x,t) = \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\left[\mathcal{N}(x,t,u_{n-1},\nu,\mu) - \mathcal{N}(x,t,u_{n-2},\nu,\mu)\right]$$

$$+\frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \left(\mathcal{N}(x,\tau,u_{n-1},\nu,\mu) - \mathcal{N}(x,\tau,u_{n-2},\nu,\mu)\right) d\tau.$$

Taking the norm of the later equation gives

$$\begin{aligned}
\|G_n(x,t)\| &= \|u_n(x,t) - u_{n-1}(x,t)\| \\
&\le \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\|\mathcal{N}(x,t,u_{n-1},\nu,\mu) - \mathcal{N}(x,t,u_{n-2},\nu,\mu)\| \\
&+ \frac{2\alpha}{(2-\alpha)M(\alpha)}\left\|\int_0^t \left[\mathcal{N}(x,\tau,u_{n-1},\nu,\mu) - \mathcal{N}(x,\tau,u_{n-2},\nu,\mu)\right] d\tau\right\| \\
&\le \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\|\mathcal{N}(x,t,u_{n-1},\nu,\mu) - \mathcal{N}(x,t,u_{n-2},\nu,\mu)\| \\
&+ \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \|\mathcal{N}(x,\tau,u_{n-1},\nu,\mu) - \mathcal{N}(x,\tau,u_{n-2},\nu,\mu)\| d\tau.
\end{aligned}$$

Using the Lipschitz condition for $\mathcal{N}$ yields

$$\|G_n(x,t)\| \le \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}L\|u_{n-1}-u_{n-2}\|+\frac{2L\alpha}{(2-\alpha)M(\alpha)}\int_0^t \|u_{n-1}-u_{n-2}\|d\tau$$

equivalent to

$$\|G_n(x,t)\| \le \frac{2(1-\alpha)L}{(2-\alpha)M(\alpha)}\|G_{n-1}\| + \frac{2L\alpha}{(2-\alpha)M(\alpha)}\int_0^t \|G_{n-1}\|d\tau. \tag{3.6}$$

The recursive's principle from (3.6) gives

$$\|G_n(x,t)\| \le \left[\left(\frac{2(1-\alpha)L}{(2-\alpha)M(\alpha)}\right)^n + \left(\frac{2L\alpha t}{(2-\alpha)M(\alpha)}\right)^n\right] u(x,0),$$

which proves that the solution exists and is continuous. To show that

$$u(x,t) = \lim_{n\to\infty} u_n(x,t)$$

is the solution of the model (3.4), we let

$$Z_n(x,t) = \bar{u}(x,t) - u_n(x,t) \qquad \text{for} \qquad n \in \mathbb{N}.$$

Hence, from (3.5), the difference $Z_n(x,t)$ between $\bar{u}(x,t)$ and $u_n(x,t)$ should tend to zero as $n \to \infty$. Indeed

$$\bar{u} - u_{n-1} = \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \left[\mathcal{N}(x,t,u,\nu,\mu) - \mathcal{N}(x,t,u_n,\nu,\mu)\right]$$
$$+ \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \left(\mathcal{N}(x,\tau,u,\nu,\mu) - \mathcal{N}(x,\tau,u_n,\nu,\mu)\right) d\tau,$$

giving

$$\begin{aligned} \|\bar{u}(x,t) - u_{n+1}\| &\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \|\mathcal{N}(x,t,u,\nu,\mu) - \mathcal{N}(x,t,u_n,\nu,\mu)\| \\ &\quad + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \|\mathcal{N}(x,\tau,u,\nu,\mu) - \mathcal{N}(x,\tau,u_n,\nu,\mu)\| d\tau \\ &\leq \frac{2L(1-\alpha)}{(2-\alpha)M(\alpha)} \|u - u_n\| + \frac{2Lt\alpha}{(2-\alpha)M(\alpha)} \int_0^t \|u - u_n\| d\tau \\ &\leq \frac{2L(1-\alpha)}{(2-\alpha)M(\alpha)} \|Z_n\| + \frac{2Lt\alpha}{(2-\alpha)M(\alpha)} \int_0^t \|Z_n\| d\tau. \end{aligned}$$

Then indeed when $n \to \infty$, then $Z_n \to 0$ and the right hand side gives

$$\lim_{n\to\infty} u_n = \bar{u}.$$

We can take $u(x,t) = \bar{u}(x,t)$ as a solution of (3.4) that is continuous. Furthermore, applying the lipschitz condition for $\mathcal{N}$, we have the following:

$$\begin{aligned} &u(x,t) - \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \mathcal{N}(x,t,u,\nu,\mu) - \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \mathcal{N}(x,t,u,\nu,\mu) d\tau \\ &= R_n(x,t) + \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \left(\mathcal{N}(x,\tau,u_{n-1},\nu,\mu) - \mathcal{N}(x,t,u,\nu,\mu)\right) \\ &+ \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \left(\mathcal{N}(x,\tau,u_{n-1},\nu,\mu) - \mathcal{N}(x,t,u,\nu,\mu)\right) d\tau. \end{aligned}$$

This yields

$$\begin{aligned} &\left\| u(x,t) - \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \mathcal{N}(x,t,u,\nu,\mu) - \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \mathcal{N}(x,t,u,\nu,\mu) d\tau \right\| \\ &= \|G_n(x,t)\| + \left(\frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} + \frac{2\theta t\alpha}{(2-\alpha)M(\alpha)} \right) \|G_{n-1}(x,t)\|. \end{aligned}$$

Passing to the limit when $n \to 0$ and considering the initial condition, we have

$$u(x,t) = u(x,0) + \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \mathcal{N}(x,t,u,\nu,\mu) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \mathcal{N}(x,t,u,\nu,\mu) d\tau.$$

For uniqueness we consider u and v be two different solutions of the model (3.4) then, the Lipschitz condition for $\mathcal{N}$ yields

$$\|u-v\| \le \frac{2L(1-\alpha)}{(2-\alpha)M(\alpha)}\|u-v\| + \frac{2Lt\alpha}{(2-\alpha)M(\alpha)}\|u-v\|$$

rearranged to be

$$\|u-v\| \left(1 - \frac{2L(1-\alpha)}{(2-\alpha)M(\alpha)} - \frac{2Lt\alpha}{(2-\alpha)M(\alpha)}\right) \le 0.$$

Then, $\|u-v\| = 0$ if

$$1 > \frac{2L(1-\alpha)}{(2-\alpha)M(\alpha)} + \frac{2Lt\alpha}{(2-\alpha)M(\alpha)}$$

and the proposition is proved. □

3.1 Numerical approximations scheme

In order to perform some simulations of the solution to the model (3.4) and compare with existing ones, we make use of the numerical approximation scheme in space and time of the new Caputo-Fabrizio fractional derivative that was recently developed in [3] where the stability and convergence analysis are discussed. In the following lines, we recall important points of the scheme. Indeed, for the finite difference scheme, we take into account a positive integer $N \in \mathbb{N}$ so as to define the grids' sizes to be $s = 1/N$ and the time grid points $t_k = ks$ taken in the time interval $[0, T]$ with $k = 0, 1, 2, \cdots M$. The function u takes the value $u^k = u(t_k)$ at the grid point t_k. Using the following formula at t_k,

$$D_t^\alpha u(t_k) = \frac{(2-\alpha)\alpha M(\alpha)}{2\,(1-\alpha)} \int_0^{t_k} \dot{u}(\tau)\, \exp\left(-\frac{\alpha}{1-\alpha}(t_k-\tau)\right) d\tau$$

and using the first order approximation $\frac{du}{dt} = \frac{u(t_{k+1})-u(t_k)}{s} + O(s)$ in

$$D_t^\alpha u(t_k) = \frac{(2-\alpha)\alpha M(\alpha)}{2\,(1-\alpha)} \times \left[\sum_{i=1}^{k} \int_{(i-1)s}^{is} \left(\frac{u^{i+1}-u^i}{k} + O(s)\right) \exp\left(-\frac{\alpha}{1-\alpha}(t_k-\tau)\right) d\tau\right],$$

the authors in [3] were able to prove the following theorem:

Theorem 1. *Let $u(: (a,b) \longrightarrow \mathbb{R}$ an arbitrary real and locally integrable function, $t_k \in (a,b)$ and α a number in $[0,1]$ then, the first order approximation of the new Caputo-Fabrizio time fractional derivative with no singular kernel at a point t_k is given by*

$$D_t^\alpha u(t_k) = \frac{M(\alpha)}{\alpha} \left[\sum_{i=1}^{k} \left(\frac{u^{i+1}-u^i}{s}\right) \theta_{i,s}\right] + O(s^2) \qquad (3.7)$$

with the coefficients

$$\theta_{i,s} = \exp\left(\frac{-\alpha s}{1-\alpha}(k-i)\right) - \exp\left(\frac{-\alpha s}{1-\alpha}(k-i+1)\right).$$

Now, applying the scheme (3.7) to the model (3.4), approximate solutions are plotted in Figure 1) for $M = 100$, $s = 0.02$ according to the initial condition $h(x) = \cos(\pi x)$ with the two perturbation parameters taking the values $\nu = 5$, $\mu = 3$ and a) $\alpha = 0.05$, b) $\alpha = 0.15$, c) $\alpha = 0.85$, d) $\alpha = 0.95$.

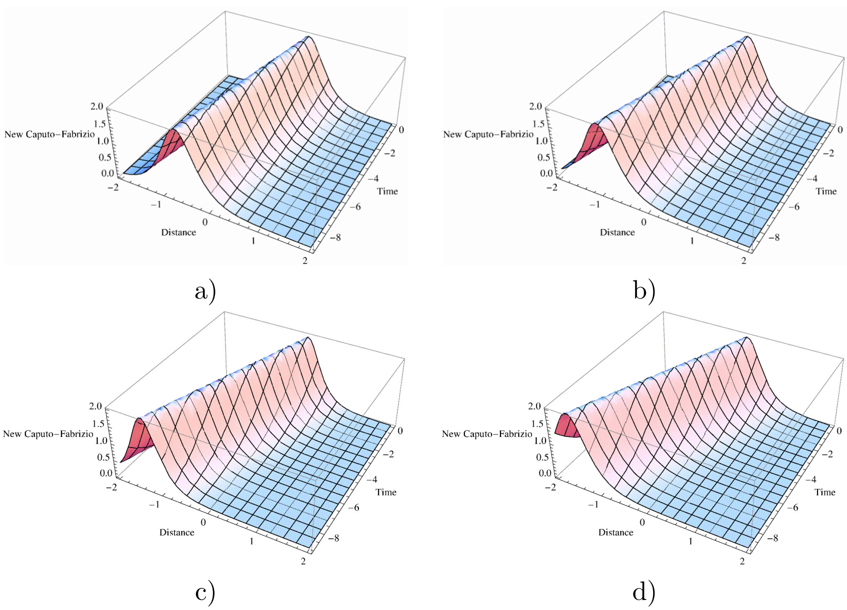

Figure 1. Solution $u(x,t)$ for $\nu = 5$, $\mu = 3$ and: a) $\alpha = 0.05$, b) $\alpha = 0.15$, c) $\alpha = 0.85$, d) $\alpha = 0.95$

4 Concluding Remarks

We have proved that it is possible to extend the analysis of the Korteweg–de Vries–Burgers equation with two perturbation's levels to the concepts of fractional differentiation, using the newly introduced Caputo-Fabrizio time fractional derivative with no singularity. We established conditions on the perturbation parameters ν, μ and the derivative order α, under which the model admits a unique solution that is continuous. Numerical approximations have been provided, clearly showing similar behavior of the solutions for closely different values of the parameters involved. This work is different from the ones presented in the previous literature and provides the new Caputo-Fabrizio derivative with a precious and promising recognition in mathematical and engineering modeling.

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