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ASYMPTOTICAL ANALYSIS OF ONE DIMENSIONAL GAS DYNAMICS EQUATIONS

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ABSTRACT

A method of averaging is developed for constructing a uniformly valid asymptotic solution for weakly nonlinear one dimensional gas dynamics systems. Using this method we give the averaged system, which disintegrates into independent equations for the non-resonance systems. Conditions of the resonance for periodic and almost periodic solutions are presented. In the resonance case the averaged system is solved numerically. Some results of numerical experiments are given.

1. INTRODUCTION

In this paper we consider the system of one dimensional Euler equations of gas dynamics [8]:

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho\\ \rho u\\ E\end{bmatrix}+\frac{\partial}{\partial x}\begin{bmatrix}\rho u\\ \rho u^2+p\\ u(E+p)\end{bmatrix}=\frac{\partial^2}{\partial x^2}\begin{bmatrix}0\\ \varepsilon\nu u\\ \varepsilon\varkappa\theta\end{bmatrix}\,, \tag{1.1}$$

where ρ is the density, u the velocity, E the total energy, θ the temperature, and p the pressure of the gas. The total energy E is decomposed as

$$E=\frac{1}{2}\rho u^2+c_v\theta\,,$$

here c_v is the specific heat at constant volume. The equation of state for a polytrophic ideal gas is given by

$$p = \mathcal{R}\rho\theta\,.$$

Equations (1.1) also include small viscosity and heat conduction terms. The simple one-dimensional case is studied in order to gain insight in the various physical and numerical problems encountered in modeling gas flows for long time intervals.

We assume that the solutions are smooth functions. Then the conservation laws (1.1) can be transformed into equivalent differential form [1]:

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial}{\partial x}(\rho u) = 0, \\ \rho\left(\dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x}\right) + \dfrac{\partial p}{\partial x} = \varepsilon\nu\dfrac{\partial^2 u}{\partial x^2}, \\ c_v\rho\left(\dfrac{\partial \theta}{\partial t} + u\dfrac{\partial \theta}{\partial x}\right) = \varepsilon\varkappa\dfrac{\partial^2 \theta}{\partial x^2} + \varepsilon\nu\left(\dfrac{\partial u}{\partial x}\right)^2 - p\dfrac{\partial u}{\partial x}\,. \end{cases} \tag{1.2}$$

Note, that during this derivation the term uu_{xx} was substituted by $-(u_x)^2$, these terms are equivalent with respect to total energy.

Let consider a constant state solution

$$\rho = \rho_0,\ u = u_0,\ \theta = \theta_0\,.$$

We are interested in a small amplitude wave solution

$$\begin{aligned} \rho(t,x) &= \rho_0 + \varepsilon\rho_1(t,x), \\ u(t,x) &= u_0 + \varepsilon u_1(t,x), \\ \theta(t,x) &= \theta_0 + \varepsilon\theta_1(t,x)\,. \end{aligned} \tag{1.3}$$

Let denote the constant state solution

$$U_0 = \begin{bmatrix} \rho_0 \\ u_0 \\ \theta_0 \end{bmatrix}.$$

Linearizing the problem (1.2) about a constant state we obtain the system

$$U_t + AU_x = \varepsilon F(U, U_x, U_{xx}) + O(\varepsilon^2) \tag{1.4}$$

with the Jacobian matrix frozen at U_0:

$$A = \begin{pmatrix} u_0 & \rho_0 & 0 \\ \dfrac{\mathcal{R}\theta_0}{\rho_0} & u_0 & \mathcal{R} \\ 0 & \dfrac{\mathcal{R}\theta_0}{c_v} & u_0 \end{pmatrix}$$

and the nonlinear interaction term:

$$F = \begin{bmatrix} -\dfrac{\partial \rho_1 u_1}{\partial x} \\ \dfrac{\nu}{\rho_0}\left(\dfrac{\partial^2 u_1}{\partial x^2} - \mathcal{R}\dfrac{\partial \rho_1 u_1}{\partial x}\right) - u_1\dfrac{\partial u_1}{\partial x} \\ \dfrac{\varkappa}{c_v\rho_0}\dfrac{\partial^2 \theta_1}{\partial x^2} - \dfrac{\mathcal{R}}{c_v}\theta_1\dfrac{\partial u_1}{\partial x} - u_1\dfrac{\partial \theta_1}{\partial x} \end{bmatrix}.$$

Usually, the term U_1, which defines the propagation of small disturbances, is determined from a simplified constant coefficient linear system [8]

$$U_t + AU_x = 0\,. \tag{1.5}$$

Higher order corrections for nonlinear problems can be obtained by retaining more terms in the expansions. Similar expansions can be used to study the propagation of discontinuities (see [10]).

The problem (1.5) is solved explicitly. First we decompose matrix A

$$A = R\Lambda R^{-1}\,,$$

where $\Lambda = diag(u_0 + \lambda_0, u_0, u_0 - \lambda_0)$ is a diagonal matrix of eigenvalues, λ_0 is the local speed of sound

$$\lambda_0 = \sqrt{\mathcal{R}\theta_0\left(\frac{\mathcal{R}}{c_v} + 1\right)}$$

and R is the matrix of right eigenvectors

$$R = \begin{pmatrix} 1 & -\dfrac{\rho_0}{\theta_0} & 1 \\ \dfrac{\lambda_0}{\rho_0} & 0 & -\dfrac{\lambda_0}{\rho_0} \\ \dfrac{\mathcal{R}\theta_0}{c_v\rho_0} & 1 & \dfrac{\mathcal{R}\theta_0}{c_v\rho_0} \end{pmatrix}.$$

Next we introduce the characteristic variables

$$V = R^{-1} U\,.$$

Multiplying (1.5) by R^{-1} gives three independent scalar equations

$$\frac{\partial v_j}{\partial t} + \lambda_j \frac{\partial v_j}{\partial x} = 0, \quad j = 1, 2, 3\,. \tag{1.6}$$

Each of these equations has a solution

$$v_j(x,t) = v_j(x - \lambda_j t, 0),$$

which describes an independently advected wave. Then the solution of problem (1.5) is the superposition of these three waves:

$$\begin{aligned}
\rho_1 &= v_1 - \frac{\rho_0}{\theta_0} v_2 + v_3\,,\\
u_1 &= \frac{\lambda_0}{\rho_0}\left(v_1 - v_3\right)\,,\\
\theta_1 &= \frac{\mathcal{R}\theta_0}{\rho_0 c_v}\left(v_1 + v_3\right) + v_2\,.
\end{aligned} \tag{1.7}$$

The obtained solution does not approximate the exact solution of Euler problem (1.2) if $t \sim \varepsilon^{-1}$. We propose to retain in the decomposed system of equations (1.6) the terms of order ε :

$$\frac{\partial v_j}{\partial t} + \lambda_j \frac{\partial v_j}{\partial x} = \varepsilon \sum_{i=1}^{3} \left(v_i \sum_{k=1}^{3} f_{jik} \frac{\partial v_k}{\partial x} + f_{ji} \frac{\partial^2 v_i}{\partial x^2} \right), \quad j = 1, 2, 3. \tag{1.8}$$

We present explicit expressions for coefficients required in the following sections

$$\begin{aligned}
f_{11} &= f_{33} = \frac{\varkappa\mathcal{R} + \nu c_v \mathcal{R} + \nu c_v^2}{2\left(\mathcal{R} + c_v\right)\rho_0 c_v},\\
f_{111} &= -f_{333} = -\frac{3\lambda_0^2 c_v^2 + \lambda_0^2 \mathcal{R}^2 + 2\lambda_0^2 c_v \mathcal{R} + \mathcal{R}^3\theta_0 - \mathcal{R}\theta_0 c_v^2}{2\rho_0 c_v \lambda_0 (\mathcal{R} + c_v)},\\
f_{132} &= -f_{312} = \frac{\mathcal{R}(\mathcal{R} - c_v)}{2\lambda_0 c_v},\\
f_{123} &= -f_{321} = \frac{\lambda_0^2 \mathcal{R} - \lambda_0^2 c_v - 2\mathcal{R}^2\theta_0 - 2\mathcal{R} c_v \theta_0}{2\theta_0\lambda_0\left(\mathcal{R} + c_v\right)}.
\end{aligned} \tag{1.9}$$

Our goal is to construct the asymptotic solution of system (1.8)

$$V_j(\tau, y_j) = v_j(t, x) + o(1)\,,$$
$$\tau = \varepsilon t, \ \ y_j = x - \lambda_j t, \ \ j = 1, 2, 3,$$

which is uniformly valid for $t \in [0, O\left(\varepsilon^{-1}\right)]$.

Systematic analysis of the proposed averaging method is given in [6; 7]. A survey of general mathematical results on asymptotical expansion methods is presented by Bhatnagar [2], Kaliakin [3], Mitropolskii [9]. This perturbation method was also used in [5] for solving the shallow water equations.

The rest of the paper is organized as follows. In section 2 we use the averaging technique and obtain the system of three equations for functions V_j. The resonance conditions are investigated in section 3. The section 4 gives numerical results for one test case when the resonance conditions are satisfied. Finally, some concluding remarks are given in section 5.

2. ASYMPTOTIC SOLUTION

Let consider the system (1.2)-(1.3) with initial conditions given in the form

$$v_j(0, x) = v_{0j}(x), \ \ j = 1, 2, 3. \tag{2.1}$$

Then we assume that the asymptotic solution $V_j(\tau, y_j)$ satisfies the same initial conditions

$$V_j(0, y_j) = v_{0j}(y_j), \ j = 1, 2, 3. \tag{2.2}$$

In order to define the differential problem for functions V_j we use the following operator of averaging along the jth characteristic of the non-perturbed (i.e., $\varepsilon = 0$) system (1.8):

$$\begin{aligned} &M_j[g(\tau, y_i, y_j, y_k)] \\ &= \lim_{T \to +\infty} \frac{1}{T} \int_0^T g(\tau, y_j + (\lambda_j - \lambda_i)s, y_j, y_j + (\lambda_k - \lambda_j)s)ds. \end{aligned} \tag{2.3}$$

Applying it to problem (1.8) we get that functions V_j must satisfy the following equations [5]

$$\frac{\partial V_j}{\partial \tau} = M_j[g_j], \ \ j = 1, 2, 3, \tag{2.4}$$

where εg_j denotes the right side of (1.8).

It is easy to prove the following properties of the averaging operators:

$$M_j[V_j\frac{\partial V_j}{\partial y_j}] \equiv V_j\frac{\partial V_j}{\partial y_j},$$
$$M_j[V_j\frac{\partial V_i}{\partial y_i}] \equiv V_j M_j[\frac{\partial V_i}{\partial y_i}] = 0, \quad i \neq j.$$

Without a loss of generality we can assume, that

$$\lim_{T\to+\infty}\frac{1}{2T}\int_{-T}^{T} v_{0j}(x)dx = 0. \tag{2.5}$$

Then we get that the averaging functions V_j satisfy the following conditions

$$M_j[V_i\frac{\partial V_j}{\partial y_j}] \;\equiv\; M_j[V_i]\frac{\partial V_j}{\partial y_j} \;=\; 0, \;\; i \neq j. \tag{2.6}$$

Now we will derive the explicit expressions of equations in system (2.4). After simple computations we have that

$$M_2[\frac{\partial V_3}{\partial y_3}V_1] = \lim_{T\to+\infty}\frac{1}{T}\int_0^T \frac{\partial V_3(\tau, y_2+\lambda_0 s)}{\partial y_2}V_1(\tau, y_2-\lambda_0 s)ds$$

and

$$\begin{aligned}
M_2[\frac{\partial V_1}{\partial y_1}V_3] &= \lim_{T\to+\infty}\frac{1}{T}\int_0^T \frac{\partial V_1(\tau, y_2-\lambda_0 s)}{\partial y_1}V_3(\tau, y_2+\lambda_0 s)ds \\
&= -\lim_{T\to+\infty}\frac{V_1(\tau, y_2-\lambda_0 T)V_3(\tau, y_2+\lambda_0 T) - V_1(\tau, y_2)V_3(\tau, y_2)}{\lambda_0 T} \\
&+ \lim_{T\to+\infty}\frac{1}{T}\int_0^T V_1(\tau, y_2-\lambda_0 s)\frac{\partial V_3(\tau, y_2-\lambda_0 s)}{\partial y_2}ds \\
&= M_2[\frac{\partial V_3}{\partial y_3}V_1]\,.
\end{aligned}$$

Thus we proved the following equality

$$M_2[\frac{\partial V_3}{\partial y_3}V_1 - \frac{\partial V_1}{\partial y_1}V_3] = M_2[\frac{\partial V_3}{\partial y_3}V_1] - M_2[\frac{\partial V_1}{\partial y_1}V_3] = 0. \tag{2.7}$$

Direct computations give coefficients f_{213} and f_{231} in system (1.8)

$$f_{213} = -f_{231} = \frac{\mathcal{R}\theta_0\lambda_0\left(\mathcal{R} - c_v\right)}{c_v\rho_0^2\left(\mathcal{R} + c_v\right)} .$$

Hence we obtain the following linear parabolic equation for the function V_2:

$$\frac{\partial V_2}{\partial \tau} = \frac{\varkappa_1}{\rho_0^2 c_v\left(\mathcal{R} + c_v\right)} \frac{\partial^2 V_2}{\partial y_2^2} \tag{2.8}$$

and it can be solved independently.

The other two functions V_1 and V_3 are determined from the following system of equations

$$\begin{cases} \dfrac{\partial V_1}{\partial \tau} = f_{111} V_1 \dfrac{\partial V_1}{\partial y_1} + f_{11} \dfrac{\partial^2 V_1}{\partial y_1^2} + M_1 \left[f_{123} V_2 \dfrac{\partial V_3}{\partial y_3} + f_{132} V_3 \dfrac{\partial V_2}{\partial y_1} \right] , \\ \dfrac{\partial V_3}{\partial \tau} = f_{333} V_3 \dfrac{\partial V_3}{\partial y_3} + f_{33} \dfrac{\partial^2 V_3}{\partial y_3^2} + M_3 \left[f_{321} V_2 \dfrac{\partial V_1}{\partial y_1} + f_{312} V_1 \dfrac{\partial V_2}{\partial y_2} \right] . \end{cases} \tag{2.9}$$

3. THE RESONANCE CONDITIONS

Let assume that in system (2.9) the averages satisfy the following conditions

$$\begin{aligned} &\lim_{T\to+\infty} \frac{1}{T} \int_0^T V_2(\tau, y_1 + \lambda_0 s) \frac{\partial V_3(\tau, y_1 + 2\lambda_0 s)}{\partial y_1} ds = 0, \\ &\lim_{T\to+\infty} \frac{1}{T} \int_0^T V_3(\tau, y_1 + 2\lambda_0 s) \frac{\partial V_2(\tau, y_1 + \lambda_0 s)}{\partial y_1} ds = 0, \\ &\lim_{T\to+\infty} \frac{1}{T} \int_0^T V_2(\tau, y_3 - \lambda_0 s) \frac{\partial V_1(\tau, y_3 - 2\lambda_0 s)}{\partial y_3} ds = 0, \\ &\lim_{T\to+\infty} \frac{1}{T} \int_0^T V_1(\tau, y_3 - 2\lambda_0 s) \frac{\partial V_2(\tau, y_3 - \lambda_0 s)}{\partial y_3} ds = 0. \end{aligned} \tag{3.1}$$

Then the averaged system (2.9) disintegrates into two independent Burgers equations:

$$\frac{\partial V_j}{\partial \tau} = f_{jjj} V_1 \frac{\partial V_j}{\partial y_j} + f_{jj} \frac{\partial^2 V_j}{\partial y_j^2}, \ j = 1, 3. \tag{3.2}$$

If conditions (3.1) are not satisfied, then system (2.9) describes the resonance interaction of waves.

Let assume that in (2.1) functions $v_{0j}(x)$ are almost periodical with periods $\mu_{jl} > 0$:

$$v_{0j}(x) = \sum_{l \neq 0} v_{0jl} e^{i\mu_{jl} x}\,, \tag{3.3}$$
$$\mu_{j(-l)} = -\mu_{jl}\,,\ j = 1, 2, 3.$$

It is easy to prove that conditions (3.1) are satisfied if for any $l_1, l_2, l_3 \in Z$, such that $|l_1| + |l_2| + |l_3| \neq 0$, we have

$$\mu_{2l_2} \neq 2\mu_{3l_3} \ \&\ \mu_{2l_2} \neq 2\mu_{1l_1}. \tag{3.4}$$

Therefore we obtained the non-resonance conditions (3.4) for almost periodical solutions.

In the resonance case all equations in the averaged system (2.9) are connected and it describes the interaction of waves. Asymptotical analysis step is finished at this stage, but some numerical analysis is still needed in order to get the solution.

4. NUMERICAL EXPERIMENTS

In this section we present results of some numerical experiments. For all tests the system (1.2) is solved with the following coefficients

$$c_v = 1,\ \ \mathcal{R} = 1,\ \ \nu = 1,\ \ \varkappa = 1.$$

Initial conditions are selected as

$$v_{01}(x) = \cos x,\ \ v_{02}(x) = \sin 2x,\ \ v_{03}(x) = \cos x. \tag{4.1}$$

Thus in equation (3.3) we have periods

$$\mu_{11} = 1,\ \ \mu_{21} = 2,\ \ \mu_{31} = 1$$

and conditions (3.4) are not satisfied. Therefore we deal with the resonance case of system (2.9).

4.1. Finite difference scheme

We define the space ω_h and time ω_τ meshes and assume that the space mesh size h and time mesh size τ are uniform. We denote by $v_j^n = v(t^n, x_j)$ a discrete function defined on $\omega_h \times \omega_\tau$. The velocity function u will be approximated at the cell faces by $u^n_{j-0.5} = u(t^n, x_{j-0.5})$.

The following common notation of difference derivatives is used in our paper

$$v_\tau = \frac{v^{n+1} - v^n}{\tau}, \quad v_{\bar{x}} = \frac{v_j - v_{j-1}}{h},$$
$$v_x = \frac{v_{j+1} - v_j}{h}, \quad v_{\overset{\circ}{x}} = \frac{v_{j+1} - v_{j-1}}{2h}.$$

Similar difference operators are defined for $u^n_{j-0.5}$.

The finite difference approximation of the Euler system (1.2) is defined as follows (see also [4; 8]):

$$\rho_\tau + \frac{(\rho u)_{i+0.5} - (\rho u)_{i-0.5}}{h} = 0\,, \tag{4.2}$$
$$(\rho u)_{i+0.5} = u^n_{i+0.5}\left(\rho_i + \frac{1}{2}\Phi(r_{i+0.5})(\rho_i - \rho_{i-1})\right)\,,$$
$$r_{i+0.5} = \frac{\rho_{i+1} - \rho_i}{\rho_i - \rho_{i-1}}\,,$$

$$c_v \rho \left(\theta_\tau + \frac{u_{i+0.5} + u_{i-0.5}}{2}\frac{\Theta_{i+0.5} - \Theta_{i-0.5}}{h}\right) = \varepsilon\varkappa\theta_{\bar{x}x} + \nu\,(u_x)^2 - p u_x\,, \tag{4.3}$$
$$\Theta_{i+0.5} = \theta_i + \frac{1}{2}\Phi(k_{i+0.5})(\theta_i - \theta_{i-1})\,,$$
$$k_{i+0.5} = \frac{\theta_{i+1} - \theta_i}{\theta_i - \theta_{i-1}}\,,$$

$$\frac{\rho^{n+1}_{i+1} + \rho^{n+1}_i}{2}\left(u_\tau + \frac{1}{2}\frac{U^2_{i+1} - U^2_i}{h}\right) + p^{n+1}_x = \varepsilon\nu u_{\bar{x}x} \tag{4.4}$$
$$U^2_{i+1} = (u^n_{i+0.5})^2 + \frac{1}{2}\Phi(l_i)\left((u^n_{i+0.5})^2 - (u^n_{i-0.5})^2\right)\,,$$
$$l_i = \frac{(u^n_{i+1.5})^2 - (u^n_{i+0.5})^2}{(u^n_{i+0.5})^2 - (u^n_{i-0.5})^2}\,.$$

Here we use the well-known limited $\kappa = \frac{1}{3}$ upwind flux approximation scheme with the special limiter [4]

$$\Phi(r) = \max\left(0, \min\left(2r, \min\left(\frac{1}{3} + \frac{2}{3}r, 2\right)\right)\right)\,.$$

The averaged system (2.9) is approximated by the following finite difference scheme:

$$V_{1,\tau} = f_{11} V_{1,\bar{y}y}^{n+1} + \frac{1}{2} f_{111} \; (V_1^{n+1})^2_{\mathring{y}} + f_{123} \; S_1(V_2, V_3) + f_{132} \; S_1(V_3, V_2) \,, \quad (4.5)$$

$$V_{2,\tau} = f_{22} V_{2,\bar{y}y}^{n+1} \,, \quad (4.6)$$

$$V_{3,\tau} = f_{33} V_{3,\bar{y}y}^{n+1} + \frac{1}{2} f_{333} \; (V_3^{n+1})^2_{\mathring{y}} + f_{312} \; S_2(V_1, V_2) + f_{321} \; S_2(V_2, V_1) \,, \quad (4.7)$$

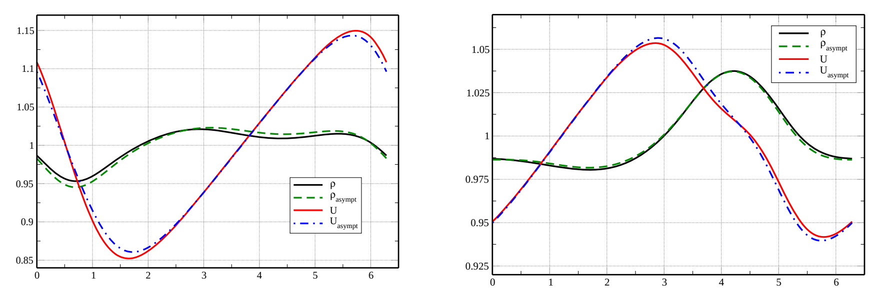
where the integrals are approximated as follows:

$$S_i(V_j, V_k) = \frac{1}{4\pi} \sum_{l=1}^{N} \Big(V_k\big(y_i + ((\lambda_i - \lambda_k)l + 1)h\big) - V_k\big(y_i + ((\lambda_i - \lambda_k)l - 1)h\big) \Big) V_j\big(y_i + ((\lambda_i - \lambda_j)l - 1)h\big) \,.$$

4.2. Simulation results

Figures 1 and 2 show the solution of system (1.2) and the asymptotic solution at $t = 1/\varepsilon$ for four different values of the small parameter ε. We present graphics of the density and velocity functions.

Note that the averaged system must be solved numerically only once and then the solution can be computed for any ε by using a simple interpolation procedure.

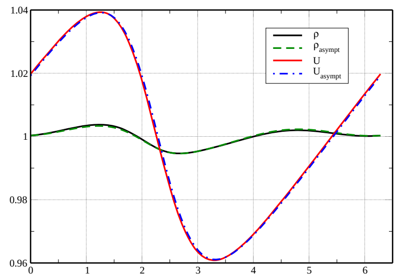


$\varepsilon = 0.1$ $\varepsilon = 0.05$

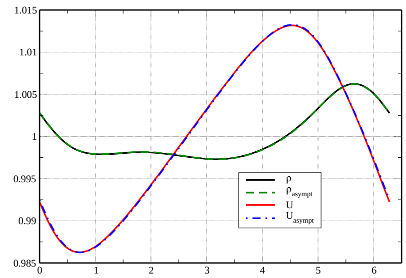
Figure 1. Asymptotical solutions of the Euler problem (1.2) for $\varepsilon = 0.1$ and $\varepsilon = 0.05$.

5. CONCLUSIONS

The method for constructing asymptotical solutions of weakly nonlinear gas dynamics equations have been presented. The solution is obtained as a superposition of three waves, which satisfy a system of nonlinear differential equations. The differential part of the system reduces to the viscous Burgers



$\varepsilon = 0.025$



$\varepsilon = 0.01$

Figure 2. Asymptotical solutions of the Euler problem (1.2) for $\varepsilon = 0.025$ and $\varepsilon = 0.01$.

equation, the interaction of waves is described by nonlinear integral terms. It is proved that this approximation is uniformly valid for time intervals of order $1/\varepsilon$.

Numerical experiments demonstrate that the proposed method can be used to solve gas dynamics problems for a broad spectrum of small parameters.

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VIENMAČIŲ DUJŲ DINAMIKOS LYGČIŲ ASIMPTOTINIS SPRENDIMO METODAS

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Darbe sukonstruotas asimptotinis skleidinys, kuris tolygiai aproksimuoja vienmačių dujų sprendinį visame intervale $t \leq O(1/\varepsilon)$. Metodika remiasi anksčiau pasiūlytu lygčių vidurkinimmo metodu. Surastos sąlygos, kada gautoji suvidurkinta diferencialinių lygčių sistema atsiskiria į tris nepriklausomas klampias Burgerso lygtis. Kai išpildytos rezonanso sąlygos, netiesinių bangų sąveika yra aprašoma integraliniais nariais. Pateiktos baigtinių skirtumų schemos, aproksimuojančios tiek pradinę diferencialinių lygčių sistemą, tiek ir suvidurkintas lygtis. Atliktas skaičiavimo eksperimentas, patvirtinantis teorines išvadas.