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THE GOURSAT PROBLEM FOR HYPERBOLIC LINEAR THIRD ORDER EQUATIONS

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ABSTRACT

The third order hyperbolic linear differential equation is considered in the non-cylindrical domain of multidimensional Euclidean space. The equation operator is a composition of a differentiation operator of the first order and second order operator, which is hyperbolic with respect to the prescribed vector field. Apart from the equation, Goursat and Cauchy conditions are defined for an unknown function. Thus the boundary of the domain, where this hyperbolic equation is defined, consists of characteristic hypersurfaces, the hypersurfaces, where Cauchy conditions are prescribed, and hypersurfaces with no conditions. For the mentioned problem the existence and uniqueness of the strong solution are proved using mollifying operators with a variable step and functional analysis methods on the base of the previously proved energy inequality.

1. INTRODUCTION

In the theory of partial differential equations there exists a formulation and decidability problem for well-defined problems. The term 'well – defined problem' was for the first time used by J. Hadamard in 1930th. [1], [2], [3]. Finding a well – defined problem, uniqueness and existence proof for its solution, and solution continuous dependence on the input data proof as well as solving the problem numerically are essential parts of creating models for natural science problems. The results of I.G. Petrovsky's research on Cauchy problem published in 1937 [4], [5] were a foundation for creating a modern theory of partial differential equations. Cauchy problem for hyperbolic equations was also considered in that work. L. Gording made an important step towards solving that problem by using functional analysis methods in a set of functional spaces [6], [7]. Many books and articles are dedicated to the solvability of mixed problems for hyperbolic equations; and the works on mixed problems in the case of cylindrical domains do much more than a half of all the works [8] – [11]. In his book [12] J.-L. Lions emphasizes the urgency and significance of exploring the problems for evolutionary type equations in the case of noncylindrical domains. Despite the first works in this sphere appeared quite a long time ago (see, for instance, the [13] – [20] and others.) many scientists still keep focusing their attention on it; and the great number of publications proves it. The mentioned works relate to a case of two independent variables or to a case of the simplest domains where the main equation or their system is defined. The interest to boundary value problems for hyperbolic equations is caused not only by the evolution of the partial differential equations theory, but also by the necessity to solve the problems which appear during the process of simulating physical and other situations in time-depending spheres. A method of energy inequalities and mollifying operators with a variable step [21 – 26] allows to solve the decidability problem for many boundary value problems for hyperbolic equations defined in noncylindrical domains with quite general configuration. A strong solution of such problem for the second order hyperbolic equations, where hyperbolicity is defined with respect to the prescribed vector field, is considered in [21]. Considering the results of this work as a base, we prove the existence and uniqueness of the solution of Goursat problem for some hyperbolic third order equations.

2. DEFINITION OF THE PROBLEM AND FUNDAMENTAL RESULTS

We consider the functions of independent variables $x = (x_1, \cdots, x_n)$, where x are elements of n – dimensional Euclidean space R^n. Remember that we consider the boundary value problem with Cauchy and Goursat conditions for the third order linear partial differential equation. The equation operator is a composition of differentiation operator $\partial/\partial \boldsymbol{p}$ and hyperbolic one over the prescribed vector field $\boldsymbol{\mathcal{N}}$ second order operator $A(x, D)$ with partial derivatives w.r.t. $x_i, i = 1, \cdots, n$.

Let $\boldsymbol{\mathcal{N}}$ be a vector field that is defined over R^n and belongs to $C^{0,}$ and let unit vectors $\boldsymbol{\eta}(x) = (\eta_1(x), \cdots, \eta_n(x))$, $|\boldsymbol{\eta}(x)|^2 = \eta_1^2(x) + \cdots + \eta_n^2(x) = 1$, be its elements. At the beginning we consider the second order linear partial differential equation

$$A(x, D)u = \sum_{i,j=1}^{n} (a_{ij}(x)u_{x_i})_{x_j} + \sum_{i=1}^{n} a_i(x)u_{x_i} + a_0(x)u, \qquad (2.1)$$

where $u_{x_i} = \frac{\partial u}{\partial x_i}$, $u_{x_i x_j} = \frac{\partial^2 u}{\partial x_i \partial x}$, $a_{ij} = a_{ji}$ $(i, j = 1, \cdots, n)$; $a_{ij}(x), a_i(x)$ are prescribed functions of independent variables x in bounded domain $Q \subset R^n$.

DEFINITION 2.1. An operator $A(x,D)$ at the point x over direction $\boldsymbol{\eta}(x)$ is called hyperbolic if the following conditions hold:

- characteristic polynomial $A_0(x,\boldsymbol{\eta}(x)) = A_0(\boldsymbol{\eta}) = \sum\limits_{i,j=1}^{n} a_{ij}(x)\eta_i\eta_j$ isn't equal to zero (in order to be exact we regard $A_0(\boldsymbol{\eta}) \geq \delta$, where δ is a positive number);
- polynomial $A_0(x,\tau\boldsymbol{\eta}(x)+\boldsymbol{\xi}(x))$ in $\tau \in R^1$ has two different real-valued solutions, where $\boldsymbol{\xi}(x) = (\xi_1(x),\dots,\xi_n(x))$, $|\boldsymbol{\xi}(x)| = 1$, $(\boldsymbol{\eta}(x),\boldsymbol{\xi}(x)) = \sum\limits_{i=1}^{n} \eta_i(x)\xi_i(x) = 0$.

DEFINITION 2.2. An operator $A(x,D)$ is said to be a hyperbolic operator in closure $\overline{Q} \subset R^n$ of domain Q if it is hyperbolic at each point $x \in \overline{Q}$ over direction $\boldsymbol{\eta}(x)$, which belongs to the vector field $\boldsymbol{\mathcal{N}}$ prescribed over $\overline{Q}$.

Suppose that a boundary ∂Q of domain Q is piecewise smooth. We'll divide the boundary surface ∂Q into classes. For this purpose we'll use characteristic polynomial, vector field $\boldsymbol{\mathcal{N}}$, and external normal vectors $\boldsymbol{\nu}(x)$ $(x \in \partial Q)$.

Let us denote by $K(x)$ the characteristic cone of differential expression (2.1) in the case of corresponding values of coefficients $a_{ij}(x)$ at the point $x \in \overline{Q}$. The set $K(x)$ is defined by a totality of vectors $\boldsymbol{\zeta}(x) = \mu(\tau\boldsymbol{\eta}(x)+\boldsymbol{\xi}(x))$, where $\mu \in [0,\infty)$, $\tau \in R^1$ and $\tau A_0(x;\boldsymbol{\eta}) \geq -A_0(x;\boldsymbol{\eta},\boldsymbol{\xi})+G_\eta^{1/2}(x;\boldsymbol{\eta},\boldsymbol{\xi})$, $G_\eta(x;\boldsymbol{\eta},\boldsymbol{\xi}) = A_0^2(x;\boldsymbol{\eta},\boldsymbol{\xi})-A_0(x;\boldsymbol{\eta})A_0(x;\boldsymbol{\xi})$, $A_0(x;\boldsymbol{\eta},\boldsymbol{\xi}) = \sum\limits_{i,j=1}^{n} a_{ij}(x)\eta_i(x)\xi_j(x)$, vectors $\boldsymbol{\eta}(x)$ and $\boldsymbol{\xi}(x)$ are the ones from Definition 2.1. Let $K^\perp(x)$ be a dual cone w.r.t $K(x)$, e.g. $K^\perp(x) = \{\boldsymbol{\chi}(x) = (\chi_1(x),\dots,\chi_n(x)) | (\boldsymbol{\chi}(x),\boldsymbol{\zeta}(x)) = \sum\limits_{i=1}^{n} \chi_i(x)\zeta_i(x) \geq 0$ for any vector $\boldsymbol{\zeta}(x) \in K(x)\}$.

Now we consider the third order differential equation in domain $Q \subset R^n$

$$\mathcal{L}u \equiv A(x;D)\frac{\partial u}{\partial \boldsymbol{\rho}} + B(x;D)u = f(x), \tag{2.2}$$

where $\partial u/\partial\boldsymbol{\rho} = \sum\limits_{i=1}^{n} \rho_i(x)\partial u/\partial x_i$ is a derivative in the vector direction $\boldsymbol{\rho}(x) = (\rho_1(x)\dots,\rho_n(x))$ of the prescribed vector field $\boldsymbol{\mathcal{P}}$, $B(x,D)u = \sum\limits_{i=1}^{n} b_i(x)\partial u/\partial x_i + b_0(x)u$.

Let $\boldsymbol{\mathcal{R}}$ be a vector field consisting of elements $\boldsymbol{r}(x) = (r_1(x),\dots,r_n(x))$ and let the following conditions be satisfied:

$(\mathcal{R}_1)$ Vector $\boldsymbol{r}(x)$ is an internal vector of the cone $K^\perp(x)$ for each point $x \in \overline{Q}$;

$(\mathcal{R}_2)$ A field $\boldsymbol{\mathcal{R}}$ belongs to C^1.

Let $\boldsymbol{\nu}(x) = (\nu_1(x),\dots,\nu_n(x))$ be a unit vector of perpendicular to surface ∂Q at the point $x \in \partial Q$. This perpendicular is external with respect to domain

Q. Denote by r_ν a scalar product $(\boldsymbol{r}(x), \boldsymbol{\nu}(x)) = \sum_{i=1}^{n} r_i(x)\nu_i(x) = r_\nu$. Suppose that ∂Q consists of the following parts:

$$\begin{aligned}
S_0 &= \{x \in \partial Q | A_0(x; \boldsymbol{\nu}(x)) \geq \delta, r_\nu(x) > 0, \delta > 0\};\\
S_1 &= \{x \in \partial Q | A_0(x; \boldsymbol{\nu}(x)) = 0, r_\nu(x) > 0, \};\\
S_2 &= \{x \in \partial Q | A_0(x; \boldsymbol{\nu}(x)) \leq -\delta, (\boldsymbol{\rho}(x), \boldsymbol{\nu}(x)) = 0\};\\
S_3 &= \{x \in \partial Q | A_0(x; \boldsymbol{\nu}(x)) = 0, r_\nu(x) < 0\};\\
S_4 &= \{x \in \partial Q | A_0(x; \boldsymbol{\nu}(x)) \geq \delta, r_\nu(x) < 0\}.
\end{aligned}$$

Note that hypersurfaces S_i $(i = 0, \dots, 4)$ are not necessarily simply connected sets!

Add the following homogeneous Goursat conditions to equation (2.2)

$$u|_{S_2} = u|_{S_3} = \frac{\partial u}{\partial \boldsymbol{\nu}}\Big|_{S_3} = 0 \tag{2.3}$$

and Cauchy conditions to S_4

$$\ell_i u = \frac{\partial_i u}{\partial \boldsymbol{p}^i}\Big|_{S_4} = \varphi_i(x), \quad i = 0, 1, 2, \tag{2.4}$$

where $\partial^0 u/\partial \boldsymbol{p}^0 = u$, $\partial/\partial \boldsymbol{p}$ is a derivative in the vector direction $\boldsymbol{p}(x) = (p_1(x), \dots, p_n(x))$ of the prescribed vector field $\boldsymbol{\mathcal{P}}$ from C^1, which is not tangent to S_4.

Treat the problem (2.2) – (2.4) as an operational equation

$$Lu = \{\mathcal{L}u, \ell_0 u, \ell_1 u, \ell_2 u\} = F, \tag{2.5}$$

L is defined over $\mathcal{D}(L) = \{u \in C^3(\overline{Q})|$ and satisfies the conditions (2.3)$\}$, where $C^3(\overline{Q})$ is a set of third order continuously differentiable functions defined over Q.

Let us define B and H spaces in order to characterize the fundamental results with respect to problem (2.2) – (2.4) strong solution existence.

Let $S(x)$ be a section of domain Q such that $S(x)$ passes through the point $x \in \overline{Q}$ and the following conditions are satisfied:

- $A_0(y, \boldsymbol{\nu}(y)) \geq \delta > 0$ for $y \in S(x)$, where $\boldsymbol{\nu}(y)$ is a unit normal vector to a surface $S(x)$ at a point $y \in S(x)$;
- $S(x)$ is a piecewise smooth hypersurface such that smooth parts of it are members of C^1;
- Two different sections from a totality of sections $\{S(x)\}_{x\in\overline{Q}}$ are mutually disjoint at each point $x \in \overline{Q}$ and all points of the first section are on the one side with respect to the second section;

- Normal vectors $\boldsymbol{\nu}(y)$ for $y \in S(x)$, which are located on the same side of hypersurface $S(x)$, all together make either acute an angle $(r_\nu(y) > 0)$ or an obtuse angle $(r_\nu(y) < 0)$ with corresponding vectors $\boldsymbol{r}_\nu(y)$.

This statement is true for all points $y \in S(x)$ of each section $S(x)$.

To each section $S(x)$ we assign the parameter t and write S^t. Let $\overline{Q} = = U_{0\leq t \leq 1} S^t$; for $t \neq \widetilde{t}$ $(t, \widetilde{t} \in [0,1])$ $S^t \bigcap S^{\widetilde{t}} = \emptyset$ ($\emptyset$ is an empty set), hypersurfaces S_0 and S_4 are members of $\{S(x)\}_{x\in\overline{Q}}$ and $S_0 = S^1,\ S_4 = S^0$.

Denote of B a Banach space so that B is obtained by enclosing the set $\mathcal{D}(L)$ with respect to the norm

$$\|u\|_B = \sup_{0\leq t\leq 1} \sum_{|\alpha|\leq 1} \left(\left\| D^\alpha \frac{\partial u}{\partial \boldsymbol{\rho}} \right\|_{L_2(S^t)} + \|D^\alpha u\|_{L_2(S^t)} \right),$$

where $\|\bullet\|_{L_2(S^t)}$ is a norm of functional set defined over hypersurface S^t. Each function of this set is Lebesgue quadratically summable.

Denote by H a Hilbert space

$$H = L_2(Q) \times H_b^2(S_H) \times H_b^1(S_H) \times H_b^0(S_H),$$

where $L_2(Q)$ is a set of functions that are Lebesgue square summable over Q. $H^i(S_4)$ is i – ordered Sobolev space of square summable functions and derivatives over S_4. $H_b^i(S_4)$ is a supplement of $\mathcal{D}(L)$ with respect to $H^i(S_4)$ space norm; and $H_b^0(S_4) = L_2(S_4)$.

Condition 1. Let $\boldsymbol{\mathcal{P}}$ be a vector space; then a scalar product $\rho_\nu(x)$ of vectors $\boldsymbol{\rho}(x)$and $\boldsymbol{\nu}(x)$ is greater than zero for each point $x \in S_1$.

Condition 2. Equation (2.2) coefficients $a_{ij}(x) \in C^1(\overline{Q})(i, j = 1, \ldots, n)$ and all the other coefficients are bounded and measurable.

Theorem 2.1. *Under Conditions 1 and 2 for problem* (2.2) – (2.4) *operator* L *we have energy inequality*

$$\|u\|_B \leq c\|Lu\|_H \tag{2.6}$$

for each $u \in \mathcal{D}(L)$. *A constant value* $c > 0$ *does not depend on* u.

Condition 3. Let Q be a domain; then it's possible to partition it into a finite number of domains using sections $S(x)$. Moreover, for every subdomain $Q_i(\bigcup_{i=1}^{i_0} \overline{Q}_i = \overline{Q})$ it's possible to select a vector field $\boldsymbol{\mathcal{R}}$ (consisting of elements $\boldsymbol{r}(x) = \{r_1(x), \ldots, r_n(x)\}$) so that the following conditions hold:

1. $(\mathcal{R}_1) - (\mathcal{R}_2)$ conditions are satisfied in the case of Q_i.
2. $r_\nu(x) = (\boldsymbol{r}(x), \boldsymbol{\nu}(x)) = 0$ for each point $x \in S_2 \bigcap \overline{Q}_i$. $\boldsymbol{\nu}(x)$ is a unit normal vector at the point $x \in S_2 \bigcap \overline{Q}_i$ with respect to Q_i.

3. Q_i is a convex set with respect to the field $\mathcal{R}$ in the following sense. The elements of $\mathcal{R}$ are uniquely determined at each point $x \in R^n$. $\mathcal{R}$ generates a totality of curves $\{r\}$, which the field is tangent to. Domain Q_i is called a convex domain with respect to $\mathcal{R}$ if Q_i can intersect every curve r, which $\mathcal{R}$ is tangent to over a simply connected set.

Condition 4. Subdomains Q_i $(i = 1, \ldots, i_0)$ are convex sets with respect to vector field $\mathcal{P}$.

Operator L allows closing $\overline{L}$ as an operator from B to H. The proof of this fact is trivial and uses just general definition of the operator closure.

DEFINITION 2.3. The solution of the operator equation

$$\overline{L}u = f, \quad u \in \mathcal{D}(\overline{L}),$$

is called a strong solution of the problem (2.2) – (2.4).

Theorem 2.2. *Suppose that conditions* 1 – 4 *and* $S_0 \bigcup S_1 \neq \emptyset$, $S_4 \bigcup S_3 \neq \emptyset$ *(where* $\emptyset$ *is an empty space) are satisfied; then for every* $F \in H$ *there exists a unique strong solution* $u \in B$ *for the problem* (2.2) – (2.4), *and*

$$\|u\|_B \leq c\|F\|_H, \tag{2.7}$$

where c *is a positive constant value.*

3. THE ENERGY INEQUALITY (THEOREM 2.1) PROOF

Each section S^t $(0 < t < 1)$ divides the domain Q into two subdomains $Q^t$and $Q^{\bar{t}}$. By Q^t we denote a subdomain such that external vector $\boldsymbol{\nu}(x)$ $(x \in S^t)$, which is normal to hypersurface S^t and makes an acute angle with vector $\boldsymbol{r}(x)$ with respect to Q^t that is scalar product $r_\nu(x) > 0$.

Then we integrate the expression $\mathcal{L}u\frac{\partial^2 u}{\partial \boldsymbol{r}\partial \boldsymbol{\rho}}$ over Q^t, where $\partial^2 u/\partial \boldsymbol{r}\partial \boldsymbol{\rho} = \sum\limits_{i,j=1}^{n} r_i(x)\frac{\partial}{\partial r_i}\left(\rho_j(x)\frac{\partial u}{\partial \rho_j}\right)$. By Cauchy – Buniakowski we have inequality

$$\left|\left(B(x,D)u, \frac{\partial^2 u}{\partial \boldsymbol{r}\partial \boldsymbol{\rho}}\right)_{L_2(Q^t)}\right| \leq c_1\Big(\sum_{|\alpha|\leq 1}\left\|D^\alpha \frac{\partial u}{\partial \boldsymbol{\rho}}\right\|^2_{L_2(Q^t)} + \sum_{|\alpha|\leq 1}\left\|D^\alpha u\right\|^2_{L_2(Q^t)}\Big), \tag{3.1}$$

where c_1 is a positive constant value. Note that the function $\partial^2 u/\partial \boldsymbol{r}\partial \boldsymbol{\rho}$ is equal to zero over S_2 and S_3, according to (2.3) conditions and by definition of vector field members $\boldsymbol{\rho}$.

Therefore by applying Theorem 3 proof from [21], (3.1) inequality and then integral Gronwall inequality [7] (I, lemma 7.1), [27] (lemma), we get the following estimate for some $c_2 > 0$ and $c_3 > 0$

$$\sum_{|\alpha|\le 1}\left\|D^\alpha\frac{\partial u}{\partial\boldsymbol{\rho}}\right\|^2_{L_2(S^t)} - c_2\sum_{|\alpha|\le 1}\left\|D^\alpha u\right\|^2_{L_2(Q^t)} \le c_3\Big(\left\|\mathcal{L}u\right\|^2_{L_2(Q)} + \sum_{i=0}^{2}\|\ell_i u\|^2_{H_b^{2-i}(S_4)}\Big). \tag{3.2}$$

If we combine the following equality

$$\sum_{|\alpha|\le 1}\frac{\partial}{\partial\overrightarrow{\rho}}\left(D^\alpha u\right)^2 = 2\sum_{|\alpha|\le 1}\sum_{i=1}^{n}D^\alpha u\frac{\partial u}{\partial x_i}D^\alpha\rho_i$$

with Condition 1, we get the inequality

$$\sum_{|\alpha|\le 1}\left\|D^\alpha u\right\|^2_{L_2(S^t)} \le c_4\left(\sum_{|\alpha|\le 1}\left\|D^\alpha\frac{\partial u}{\partial\boldsymbol{\rho}}\right\|^2_{L_2(Q^t)} + \sum_{|\alpha|\le 1}\left\|D^\alpha u\right\|^2_{L_2(Q^t)}\right).$$

Adding it to (3.2) and applying Gronwall inequality, we finally have

$$\sum_{|\alpha|\le 1}\Big(\left\|D^\alpha\frac{\partial u}{\partial\boldsymbol{\rho}}\right\|_{L_2(S^t)} + \|D^\alpha u\|_{L_2(S^t)}\Big) \le c_5\big(\|\mathcal{L}u\|_{L_2(Q)} + \sum_{i=0}^{2}\|\ell_i u\|_{H_b^{2-i}(S_4)}\big)$$

and energy inequality, which is being proved, easily follows it.

Passing to the limit in (2.6) we obtain energy inequality for operator $\overline{L}$

$$\|u\|_B \le c\|\overline{L}u\|_H \tag{3.3}$$

Inequality (3.3) is true for any function u from range of definition $\mathcal{D}(\overline{L})$ for operator $\overline{L}$.

4. THEOREM 2.2 PROOF

Since the problem (2.2) is linear, it follows that the problem strong solution is unique. Moreover, it satisfies the estimate (2.7) in case it exists. Taking into account a general theory of closure operators, we obtain that to prove the existence problem's (2.2) – (2.4) strong solution for every $F \in H$ we just have to prove that the set $\mathcal{R}(L)$ of operator L values is dense in H [28], [29]. Furthermore, by extending the parameter method (see Theorem proved in [29]), it's sufficient to prove that the set $\mathcal{R}(L_0)$ of operator $L_0 = \{\mathcal{L}_0, \ell_0, \ell_1, \ell_2\}$

values is dense in H, where operator L_0 range of definition $\mathcal{D}(L_0)$ is equal to $\mathcal{D}(L)$, $\mathcal{L}_0 = A_0(x,D)\partial/\partial\boldsymbol{\rho}$,

$$A_0(x,D) = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_j}\Big(a_{ij}(x)\frac{\partial}{\partial x_i}\Big).$$

Let the equality

$$(A_0\frac{\partial}{\partial\boldsymbol{\rho}},\vartheta)_{L_2(Q)} + (\ell_0 u, v_0)_{H_b^2(S_4)} + (\ell_1 u, v_1)_{H_b^1(S_4)} + (\ell_2 u, v_2)_{L_2(S_4)} = 0 \tag{4.1}$$

be true for a certain element $\vartheta = \{v(x), v_0(x), v_1(x), v_2(x)\}$ and for each $u \in \mathcal{D}(L_0)$. Supposing u to be any element of $\mathcal{D}_0(L_0) = \{u \in \mathcal{D}(L_0)|\ \ell_0 u = = \ell_1 u = \ell_2 u = 0\}$, in (4.1) we get

$$\Big(A_0\frac{\partial u}{\partial\boldsymbol{\rho}},\vartheta\Big)_{L_2(Q)} = 0 \tag{4.2}$$

for every $u \in \mathcal{D}_0(L_0)$. According to Theorem 2 conditions, we get that both derivatives $\omega(x) = \partial u/\partial\boldsymbol{\rho}$ over $S_2\bigcup S_3\bigcup S_4$ and $\partial\omega/\partial\boldsymbol{p}$ over S_4 are equal to zero for each $u \in \mathcal{D}(L_0)$. Therefore the equality (4.2) can be considered as orthogonal condition for element $\vartheta \in L_2(Q)$ and operator A_0 set of values $A_0\omega$ for every $\omega \in \mathcal{D}(A_0) = \{\omega \in C^3(\overline{Q})|\ \omega = 0$ over $S_2\bigcup S_3\bigcup S_4$ and $\partial\omega/\partial\boldsymbol{\nu} = 0$ over $S_4\}$. Thus as it follows from Theorem 4 proof in [21], we have (4.2) for the case $\vartheta = 0$ over $L_2(Q)$.

Turn back to equality (4.1). $\ell_i u$ $(i = 0,1,2)$ are linearly independent and each operator ℓ_i set of values $\mathcal{R}(\ell_i)$ makes a dense set in corresponding space $H_b^{2-i}(S_4)$ in case u runs through the whole range of definition $\mathcal{D}(L_0)$. Thus we have that (4.1) is an equality for each $u \in (L_0)$ if and only if all v_i $(i = 0,1,2)$ are equal to zero. We see that v is a null element of H. This completes the proof of Theorem 2.2.

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Trečios eilės tiesinių hiperbolinių lygčių Goursat uždavinys

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Daugiamatė Euklido erdvės necilindrinėje srityje nagrinėjama trečios eilės tiesinė hiperbolinė lygtis. Lygties operatorius yra pirmos eilės diferencialinio operatoriaus ir antros eilės operatoriaus, kuris yra hiperbolinis apibrėžto vektorinio lauko atžvilgiu, kompozicija. Srities kontūrą sudaro charakteristinis hiperpaviršius (jame formuojama Goursat sąlyga), hiperpaviršiaus, kuriame formuluojama Caushy sąlyga, ir laisvas nuo bet kokių sąlygų hiperpaviršius. Naudojantis kintamojo žingsnio suvidurkinto operatoriaus bei funkcinės analizės metodais, paremtais energetine nelygybe, įrodytas šio uždavinio stipriojo sprendinio egzistavimas ir vienatis.