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MULTIMOORA FOR THE EU MEMBER STATES UPDATED WITH FUZZY NUMBER THEORY

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Abstract. Fuzzy logic handles vague problems in various areas. The fuzzy numbers can represent either quantitative or qualitative variables. The quantitative fuzzy variables can embody crisp numbers, aggregates of historical data or forecasts. The qualitative fuzzy variables may be applied when dealing with ordinal scales. The MULTIMOORA method (Multiplicative and Multi-Objective Ratio Analysis) was updated with fuzzy number theory. The MULTIMOORA method consists of three parts, namely Ratio System, Reference Point and Full Multiplicative Form. Accordingly, each of them was modified with triangular fuzzy number theory. The fuzzy MULTIMOORA summarizes the three approaches. The problem remains how to summarize them. It cannot be done by summation as they are composed of ranks (ordinal). Indeed summation of ranks is against any mathematical logic. Another method, the Dominance Method, is used to rank the EU Member States according to their performance in reaching the indicator goals of the Lisbon Strategy 2000–2008. This ranking will group the best performing countries in a Core Group, followed by a Second Group, the Semi-periphery Group. Group 3, the Periphery Group, will encompass the less performing states.

Keywords: multi-objective optimization, MULTIMOORA, fuzzy number theory, structural indicators, Lisbon strategy, European Union, dominance, core, semi-periphery, periphery.

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JEL classification: C44, C61, E61, F53, N14.

1. Introduction

Multi-Objective Optimization (MOO) methods deal with problems of compromise selection of the best solutions from the set of available alternatives $\mathbf{A} = \left\{A_1; A_2; \ldots; A_j; \ldots; A_n\right\}$

according to objectives $\mathbf{C} = \{C_1; C_2; \ldots; C_i; \ldots C_m\}$. Usually neither of the alternatives satisfies all the objectives therefore *satisfactory* decision is made instead of *optimal* one. Roy (1996) presented the following pattern of MOO problems: 1) α *choosing* problem – choosing the best alternative from $\mathbf{A}$; 2) β *sorting* problem – classifying alternatives of $\mathbf{A}$ into relatively homogenous groups; 3) γ *ranking* problem – ranking alternatives of $\mathbf{A}$ from best to worst; 4) δ *describing* problem – describing alternatives of $\mathbf{A}$ in terms of their peculiarities and features. Hence, during last few decades there were many Multi- Objective methods developed. Usually MOO techniques are classified into multiple objective decision making (MODM) and multiple attribute decision making (MADM). While MODM deals with continuous optimization problems and virtually infinite set of alternatives, MADM methods are aimed at discrete optimization and finite set of pre-defined alternatives. In this article term MOO will refer to MADM. The MOO methodology and methods were overviewed by Guitouni and Martel (1998) and Zavadskas *et al.* (2008b). Kaplinski (2009) presented an overview of advances in MOO science.

The MOO procedure usually consists of three basic stages: 1) identification of alternatives; 2) selection of objectives or indicators; 3) the choice of the problem with the appropriate MOO method (Roy 2005). Whereas the first stage is quite unequivocal the remaining two could raise some questions. Objectives can encompass non-subjective as well as subjective attributes (Liang, Wang 1991; Heragu 1997; Chou *et al.* 2008). Non-subjective indicators (attributes) are quantitative, e.g., investment costs. Subjective indicators are qualitative such as stakeholders' opinions. Therefore, decision making often relies on complex as well as on vague issues. Zadeh, the Founder of fuzzy logic (1965), proposed employing the fuzzy set theory as a modeling tool for complex systems that are hard to define exactly in crisp numbers. Fuzzy logic hence allows coping with vague, imprecise and ambiguous input and knowledge (Kahraman 2008; Kahraman and Kaya 2010). Linguistic variables expressed in fuzzy numbers were introduced by Zadeh (1975a, 1975b, 1975c) and applied in many studies (Liang 1999; Chen 2000; Chou *et al.* 2008; Torlak *et al.* 2011). Grey numbers were also applied in the decision making branch (Zavadskas *et al.* 2008a, 2008c; Lin *et al.* 2008; Zavadskas *et al.* 2010a; Peldschus *et al.* 2010) when creating MOO methods suitable for fuzzy inputs[1].

The question of extending the existing MOO methods to the fuzzy environment is of high importance. The Analytic Hierarchy Process (AHP) was initially proposed by Saaty (1980) and extended into fuzzy environment (van Laarhoven, Pedrycz 1983; Leung, Cao 2000). The simple additive weight (SAW) method (MacCrimmon 1968) was updated with fuzzy numbers theory and integrated with other decision making techniques (Chou *et al*. 2008). Technique for the Order Preference by Similarity to Ideal Solution (TOPSIS) was introduced by Hwang and Yoon (1981) and updated with fuzzy number theory (Chen 2000; Liu 2009a; Zavadskas and Antucheviciene 2006). The Method of Complex Proportional Assessment (COPRAS) (Zavadskas *et al.* 1994) was improved by applying fuzzy number technique (Zavadskas, Antuchevičienė 2007). Zavadskas and Turskis introduced another method ARAS (2010), extended with grey and triangular fuzzy number (Turskis and Zavadskas 2010a, 2010b). Liang and Ding (2003) developed fuzzy MOO method based on α-cut concept. Peldschus

[1] Mukaidono (2001) presents an interesting introduction to fuzzy logic. Zopounidis *et al*. (2001) with "Fuzzy sets in Management, Economics and Marketing" are perhaps nearer to the topic of this article.

and Zavadskas (2005) applied fuzzy game theory in multiple objective evaluation. Hence, updating MOO methods with fuzzy number theory is important.

Brauers and Zavadskas (2006) introduced Multi-Objective Optimization by Ratio Analysis (MOORA) on basis of previous research by Brauers (2004). In 2010 these authors developed this method further which became MULTIMOORA (MOORA plus the full multiplicative form). Numerous examples of application of these methods are present (Brauers *et al.* 2007, 2008, 2010; Brauers and Ginevičius 2009, 2010; Brauers and Zavadskas 2009a, 2009b; Baležentis and Baležentis 2010; Baležentis *et al.* 2010; Chakraborty 2010). However MULTIMOORA has not been updated with fuzzy numbers theory yet. This article deals with the issue of updating MULTIMOORA method with triangular fuzzy number theory and applying the fuzzy MULTIMOORA in international comparison of the European Union Member States.

The article is therefore organized in the following way. Section 2 deals with fuzzy set theory. The following Section 3 focuses on MULTIMOORA method. The proposed fuzzy MULTIMOORA method is described in Section 4. Section 5 undertakes a numerical example where the European Union (EU) Member States are compared on a basis of structural indicators and the new method. The data covers the period of 2000–2008. Section 6 makes a distinction between cardinal and ordinal scales in MULTIMOORA. Section 7 brings the application of the Multi-Objective Optimization on the European Union Member States based on MULTIMOORA.

2. The fuzzy set theory and triangular fuzzy numbers

Fuzzy sets and fuzzy logic are powerful mathematical tools for modeling uncertain systems. A fuzzy set is an extension of a crisp set. Crisp sets only allow full membership or non-membership, while fuzzy sets allow partial membership. The theoretical fundaments of fuzzy set theory are overviewed by Chen (2000).

In a universe of discourse *X*, a fuzzy subset $\tilde{A}$ of *X* is defined with a membership function $\mu_{\tilde{A}}(x)$ which maps each element $x \in X$ to a real number in the interval [0; 1]. The function value of $\mu_{\tilde{A}}(x)$ resembles the grade of membership of *x* in $\tilde{A}$. The higher the value of $\mu_{\tilde{A}}(x)$, the higher the degree of membership of *x* in $\tilde{A}$ (Keufmann and Gupta 1991). Noteworthy, in this study any variable with tilde will denote a fuzzy number.

A fuzzy number $\tilde{A}$ is described as a subset of real number whose membership function $\mu_{\tilde{A}}(x)$ is a continuous mapping from the real line $\Re$ to a closed interval [0; 1], which has the following characteristics: 1) $\mu_{\tilde{A}}(x) = 0$, for all $x \in (-\infty; a] \cup [c; \infty)$; 2) $\mu_{\tilde{A}}(x)$ is strictly increasing in [a; b] and strictly decreasing in [d; c]; 3) $\mu_{\tilde{A}}(x) = 1$, for all $x \in [b; d]$, where *a*, *b*, *d*, and *c* are real numbers, and $-\infty < a \le b \le d \le c < \infty$. When *b* = *d* a fuzzy number $\tilde{A}$ is called a triangular fuzzy number (Fig. 1) represented by a triplet (a, b, c).

Triangular fuzzy numbers will therefore be used in this study to characterize the alternatives. The membership function $\mu_{\tilde{A}}(x)$ is thus defined as:

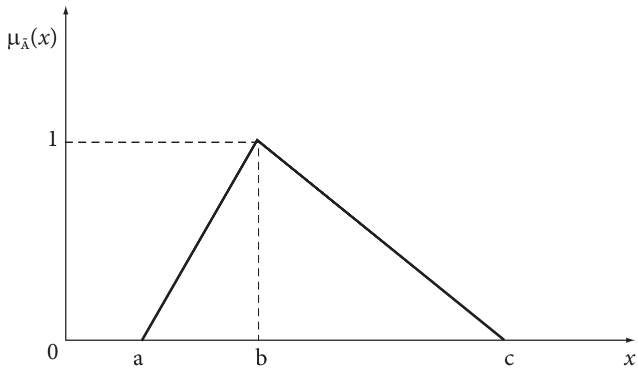

Fig. 1. Membership function of a triangular fuzzy number $\tilde{A} = (a,b,c)$.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a, \\ \dfrac{x-a}{b-a}, & a \le x \le b, \\ \dfrac{x-c}{b-c}, & b \le x \le c, \\ 0, & x > c. \end{cases} \tag{1}$$

In addition, the parameters *a*, *b*, and *c* in (1) can be considered as indicating respectively the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event (Torlak *et al.* 2011).

Let $\tilde{A}$ and $\tilde{B}$ be two positive fuzzy numbers (Liang, Ding 2003). Hence, the main algebraic operations of any two positive fuzzy numbers $\tilde{A} = (a,b,c)$ and $\tilde{B} = (d,e,f)$ can be defined in the following way (Zavadskas, Antuchevičienė 2007):

1. Addition $\oplus$:

$$\tilde{A} \oplus \tilde{B} = (a,b,c) \oplus (d,e,f) = (a+d, b+e, c+f). \tag{2}$$

2. Subtraction $\ominus$:

$$\tilde{A} \ominus \tilde{B} = (a,b,c) \ominus (d,e,f) = (a-f, b-e, c-d). \tag{3}$$

3. Multiplication $\otimes$:

$$\tilde{A} \otimes \tilde{B} = (a,b,c) \otimes (d,e,f) = (a \times d, b \times e, c \times f). \tag{4}$$

4. Division $\oslash$:

$$\tilde{A} \oslash \tilde{B} = (a,b,c) \oslash (d,e,f) = (a \setminus f, b \setminus e, c \times d). \tag{5}$$

The vertex method will be applied to measure the distance between two fuzzy numbers. Let $\tilde{A} = (a,b,c)$ and $\tilde{B} = (d,e,f)$ be two triangular fuzzy numbers. Then, the vertex method can be applied to measure the distance between these two fuzzy numbers:

$$d(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{3}[(a-d)^2 + (b-e)^2 + (c-f)^2]}. \tag{6}$$

Fuzzy numbers can be applied in two ways when forming the response matrix of alternatives on objectives. First, fuzzy numbers can represent the values of linguistic variables (Zadeh 1975a, 1975b, 1975c) when deciding either on the importance of criteria or performing qualitative evaluation of alternatives. For the latter purpose Chen (2000) describes the following fuzzy numbers identifying values of linguistic variables from scale Very poor to Very good: Very poor – (0, 0, 1); Poor – (0, 1, 3); Medium poor – (1, 3, 5); Fair – (3, 5, 7); Medium good – (5, 7, 9); Good – (7, 9, 10); Very good – (9, 10, 10). Second, the fuzzy numbers can represent monetary (quantitative) terms. It can be done either through direct input of certain fuzzy numbers into the response matrix or by aggregation of raw data (e. g. time series). For example, if there are costs "approximately equal to $200" estimated, the sum can be represented by triangular fuzzy number (190, 200, 210). Moreover, the fuzzy numbers can embody expected rate of growth. For example, if there is level of unemployment of 5 per cent with expected growth of 10 per cent, a triangular fuzzy number (5, 5.5, 6.1) can summarize these characteristics. As for time series data, a fuzzy number can represent the dynamics of certain indicator during past *t* periods:

$$\left(\min_p\{a_p\},\left(\prod_{p=1}^{t}a_p\right)^{1/t},\max_p\{a_p\}\right), \tag{7}$$

where a_p represents the value of certain indicator during period p ($p = 1,2,\ldots,t$).

The results of comparison of alternatives based on fuzzy numbers are also expressed in fuzzy numbers. The fuzzy numbers therefore need to be converted into crisp ones in order to identify the most promising alternative. There are four defuzzification methods commonly employed: (i) the centered method (or centre of area – COA); (ii) the Mean-of-maximum (MOM); (iii) the α-cut method; and (iv) the signed distance method (Zhao and Govind 1991; Yao and Wu 2000). In this study the COA method will be applied to obtain the Best Non-fuzzy Performance (BNP) value:

$$BNP_{\tilde{A}} = \frac{(c-a)+(b-a)}{3} + a, \tag{8}$$

where a, b and c are respectively the lower, modal, and upper values of fuzzy number $\tilde{A} = (a,b,c)$ [2] (Triantaphyllou 2000; Zavadskas and Antucheviciene 2006). Moreover, the robustness as well as precision of multi–criteria optimization can be improved by applying either intuitionist fuzzy numbers (Zhang, Liu 2010) or two–tuple linguistic representation (Liu 2009b).

3. The MULTIMOORA method

As already said earlier, Multi-Objective Optimization by Ratio Analysis (MOORA) method was introduced by Brauers and Zavadskas (2006) on the basis of previous research (Brauers 2004). Brauers, Zavadskas (2010) and Brauers, Ginevičius (2010) extended the method and

[2] Mode is the measurement with the maximum frequency if there is one. As there is only a lower limit and an upper limit the average of both is taken.

in this way it became more robust as MULTIMOORA (MOORA plus the full multiplicative form). These methods have been applied in numerous studies (Brauers *et al.* 2007, 2010; Brauers, Ginevičius 2009; Brauers, Zavadskas 2009a, 2009b; Brauers, Ginevičius 2010; Baležentis *et al.* 2010) focused on regional studies, international comparisons and investment management.

MOORA method begins with matrix X where its elements x_{ij} denote i^{th} alternative of j^{th} objective ($i=1,2,\cdots,m$ and $j=1,2,\cdots,n$). MOORA method consists of two parts: the ratio system and the reference point approach. MacCrimmon (1968) defines two stages of *weighting*, namely *normalization* and voting on *significance* of objectives. The issue of weighting is discussed by Brauers, Zavadskas (2010); Zavadskas *et al.* (2010b), while the problem of normalization is analyzed by Brauers (2007) and Turskis *et al.* (2009). The MULTIMOORA method includes internal normalization and treats originally all the objectives equally important. In principle all stakeholders interested in the issue only could give more importance to an objective. Therefore they could either multiply the dimensionless number representing the response on an objective with a significance coefficient or they could decide beforehand to split an objective into different sub-objectives (Brauers, Ginevičius 2009).

The Ratio System of MOORA. Ratio system defines data normalization by comparing alternative of an objective to all values of the objective:

$$x_{ij}^{*}=\frac{x_{ij}}{\sqrt{\sum_{i=1}^{m}x_{ij}^{2}}}, \tag{9}$$

where x_{ij}^{*} denotes i^{th} alternative of j^{th} objective (in this case j^{th} structural indicator of i^{th} state). Usually these numbers belong to the interval [-1; 1]. These indicators are added (if desirable value of indicator is maxima) or subtracted (if desirable value is minima) and summary index of state is derived in this way:

$$y_{i}^{*}=\sum_{j=1}^{g}x_{ij}^{*}-\sum_{j=g+1}^{n}x_{ij}^{*}, \tag{10}$$

where $g=1,\cdots,n$ denotes number of objectives to be maximized. Then every ratio is given the rank: the higher the index, the higher the rank.

The Reference Point of MOORA. Reference point approach is based on the Ratio System. The Maximal Objective Reference Point (vector) is found according to ratios found in formula (9). The j^{th} coordinate of the reference point can be described as $r_{j}=\max_{i}x_{ij}^{*}$ in case of maximization. Every coordinate of this vector represents maxima or minima of certain objective (indicator). Then every element of normalized responses matrix is recalculated and final rank is given according to deviation from the reference point and the Min-Max Metric of Tchebycheff:

$$\min_{i}\left(\max_{j}\left|r_{j}-x_{ij}^{*}\right|\right). \tag{11}$$

The Full Multiplicative Form and MULTIMOORA. Brauers and Zavadskas (2010) proposed MOORA to be updated by the Full Multiplicative Form method embodying maximization as well as minimization of purely multiplicative utility function. Overall utility of the i^{th} alternative can be expressed as dimensionless number:

$$U_i^{'} = \frac{A_i}{B_i}, \tag{12}$$

where $A_i = \prod_{j=1}^{g} x_{ij},\ i = 1,2,\cdots,m$ denotes the product of objectives of the i^{th} alternative to be maximized with $g = 1,\cdots,n$ being the number of objectives (structural indicators) to be maximized andwhere $B_i = \prod_{j=g+1}^{n} x_{ij}$ denotes the product of objectives of the i^{th} alternative to be minimized with $n - g$ being the number of objectives (indicators) to be minimized. Thus MULTIMOORA summarizes MOORA (i.e. Ratio System and Reference point) and the Full Multiplicative Form. Ameliorated Nominal Group and Delphi techniques can also be used to reduce remaining subjectivity (Brauers and Zavadskas 2010).

4. The fuzzy MULTIMOORA method

The fuzzy MULTIMOORA begins with response matrix $\tilde{X}$ with $\tilde{x}_{ij} = (x_{ij1}, x_{ij2}, x_{ij3})$ being the i^{th} alternative of the j^{th} objective ($i = 1,2,\cdots,m$ and $j = 1,2,\cdots,n$).

4.1. The fuzzy Ratio System

The Ratio System defines normalization of the fuzzy numbers $\tilde{x}_{ij}$ resulting in matrix of dimensionless numbers. The normalization is performed by comparing appropriate values of fuzzy numbers:

$$\tilde{x}_{ij}^{*} = \left(x_{ij1}^{*}, x_{ij2}^{*}, x_{ij3}^{*}\right) = \left(\frac{x_{ij1}}{\sqrt{\sum_{i=1}^{m} x_{ij1}^{2}}}, \frac{x_{ij2}}{\sqrt{\sum_{i=1}^{m} x_{ij2}^{2}}}, \frac{x_{ij3}}{\sqrt{\sum_{i=1}^{m} x_{ij3}^{2}}} \right), \forall i, j. \tag{13}$$

The normalization is followed by computation of summarizing ratios $\tilde{y}_i^{*}$ for each i^{th} alternative. The normalized ratios are added or subtracted according to formulas (2) or (3) respectively:

$$\tilde{y}_i^{*} = \sum_{j=1}^{g} \tilde{x}_{ij}^{*} \ominus \sum_{j=g+1}^{n} \tilde{x}_{ij}^{*}, \tag{14}$$

where $g = 1,2,\ldots,n$ stands for number of indicators to be maximized. Then each ratio $\tilde{y}_i^{*} = (y_{i1}^{*}, y_{i2}^{*}, y_{i3}^{*})$ is de-fuzzified by applying Eq. 8:

$$BNP_i = \frac{(y_{i3}^{*} - y_{i1}^{*}) + (y_{i2}^{*} - y_{i1}^{*})}{3} + y_{i1}^{*}, \tag{15}$$

where BNP_i denotes the best non-fuzzy performance value of the i^{th} alternative. Consequently, the alternatives with higher BNP values are attributed with higher ranks.

4.2. The fuzzy Reference Point

The fuzzy Reference Point approach is based on the fuzzy Ratio System. The Maximal Objective Reference Point (vector) $\tilde{r}$ is found according to ratios found in formula (13). The

j^{th} coordinate of the reference point resembles the fuzzy maxima or minima of j^{th} criterion $\tilde{x}_j^+$, where

$$\begin{cases} \tilde{x}_j^+ = \left(\max_i x_{ij1}^*, \max_i x_{ij2}^*, \max_i x_{ij3}^* \right), j \le g; \\ \tilde{x}_j^+ = \left(\min_i x_{ij1}^*, \min_i x_{ij2}^*, \min_i x_{ij3}^* \right), j > g. \end{cases} \tag{16}$$

Then every element of normalized responses matrix is recalculated and final rank is given according to deviation from the reference point (Eq. 6) and the Min-Max Metric of Tchebycheff:

$$\min_i \left(\max_j d(\tilde{r}_j, \tilde{x}_{ij}^*) \right). \tag{17}$$

4.3. The fuzzy Full Multiplicative Form

Overall utility of the i^{th} alternative can be expressed as dimensionless number by employing Eq. 5:

$$\tilde{U}_i^{'} = \tilde{A}_i \oslash \tilde{B}_i, \tag{18}$$

where $\tilde{A}_i = (A_{i1}, A_{i2}, A_{i3}) = \prod_{j=1}^{g} \tilde{x}_{ij}$, $i = 1, 2, \cdots, m$ denotes the product of objectives of the i^{th} alternative to be maximized with $g = 1, \cdots, n$ being the number of objectives (structural indicators) to be maximized and where $\tilde{B}_i = (B_{i1}, B_{i2}, B_{i3}) = \prod_{j=g+1}^{n} \tilde{x}_{ij}$ denotes the product of objectives of the i^{th} alternative to be minimized with $n - g$ being the number of objectives (indicators) to be minimized. Formula (4) is applied when computing these variables. Since overall utility $\tilde{U}_i^{'}$ is fuzzy number, Eq. 8 has to be used to rank the alternatives. The higher the BNP, the higher the rank of certain alternative.

Thus fuzzy MULTIMOORA summarizes fuzzy MOORA (i. e. fuzzy Ratio System and fuzzy Reference Point) and the fuzzy Full Multiplicative Form.

5. A comparison of the European Union Member States according to fuzzy MULTIMOORA

The fuzzy MULTIMOORA was applied when comparing EU Member States. Empirical analysis of EU Member States' efforts in seeking Lisbon goals began with definition of system of structural indicators (Table 1). The system consists of 12 indicators from the shortlist of structural indicators. Directions of optimization were also attributed to each of the indicator. For example, rising level of unemployment has negative economic and social consequences (Martinkus *et al.* 2009; Korpysa 2010) therefore it should be minimized.

The indicators are measured in different dimensions. The volume index of GDP per capita in Purchasing Power Standards (PPS) is expressed in relation to the European Union (EU–27) average set to equal 100. If the index of a country is higher than 100, this country's level of GDP per head is higher than the EU average and vice versa. Labor productivity per person employed is measured as GDP in PPS per person employed relative to EU-27 average (EU-27 = 100). The employment rate is calculated by dividing the number of persons aged 15 to 64 in employment by the total population of the same age group. The employment rate of older workers is calculated by dividing the number of persons aged 55 to 64 in employment by the total population of the same age group. The indicator *Youth education attainment level* is defined as the percentage of young people aged 20–24 years having attained at least upper secondary education attainment level. Gross domestic expenditure on R&D is expressed as a percentage of GDP. Business investment is defined as total gross fixed capital formation expressed as a percentage of GDP, for the private sector. Comparative price levels are the ratio between Purchasing power parities and market exchange rate for each country shown in relation to the EU average (EU-27=100). The share of persons with an equivalised disposable income below the risk-of-poverty threshold, which is set at 60% of the national median equivalised disposable income (after social transfers) is resembled by *At-risk-of-poverty rate* indicator. Long–term unemployment rate is number of persons that have been unemployed for more than 12 months expressed as the percentage of total labor force. Greenhouse gas emissions indicator presents annual total emissions (CO2 equivalents) in relation to "Kyoto base year". In general the base year is 1990 for the non-fluorinated gases and 1995 for the fluorinated gases. Gross inland consumption of energy divided by GDP (kilogram of oil equivalent per 1000 Euro) results in the *Energy intensity of the economy* indicator. However, the application of MULTIMOORA method enables to summarize all these indicators expressed in different dimensions.

Table 1. System of structural indicators used in analysis of EU Member States' development during 2000–2008

	Structural indicators	Desirable values
	I. General economic background	
1	GDP per capita in PPS (EU-27 = 100)	Max
2	Labor productivity per person employed	Max
	II. Employment	
3	Employment rate	Max
4	Employment rate of older workers	Max
	III. Innovation and research	
5	Youth education attainment level	Max
6	Gross domestic expenditure on R&D	Max
	IV. Economic reform	
7	Business investment	Max
8	Comparative price levels	Min
	V. Social cohesion	
9	At-risk-of-poverty rate	Min
10	Long-term unemployment rate	Min
	VI. Environment	
11	Greenhouse gas emissions	Min
12	Energy intensity of the economy	Min

Data covering these indicators and the period 2000–2008 were obtained from EUROSTAT Structural Indicators database and are available from the authors upon request. Due to limited data availability three time points were chosen for analysis, namely years 2000, 2004 and 2008. The data therefore cover 27 Member States, 3 years and 12 structural indicators, 972 observations in total.

The initial data (Annex A, Table A1) were aggregated by employing Eq. 8. Minimal values, geometric means and maximum values (denoted as *min*, *average* and *max* respectively in Table A2, Annex A) were obtained for each indicator thus creating the fuzzy response matrix $\widetilde{X}$ (Table A2) containing 324 fuzzy numbers. The data were internally normalized by applying Eq. 13: each response x_{ijk}, $k = 1,2,3$, was divided by respective ratio presented in the last row of Table A2 (Annex A). Hence the fuzzy normalized response matrix $\tilde{X}^*$ was formed (Table A3, Annex A).

Aggregation of normalized fuzzy ratios was performed according to Eq. 14. In this way the summarizing fuzzy ratios $\tilde{y}_i^* = \left(y_{i1}^*, y_{i2}^*, y_{i3}^*\right)$ were obtained and de-fuzzified by applying Eq. 15:

$$BNP_i = \frac{(y_{i3}^* - y_{i1}^*) + (y_{i2}^* - y_{i1}^*)}{3} + y_{i1}^* . \qquad (19)$$

BNP expressed in crisp numbers enabled to attribute each EU Member State with appropriate rank (Table A4, Annex A).

The fuzzy Reference Point relies on ratios retrieved by fuzzy Ratio System. Table A5a (Annex A) presents the coordinates of fuzzy vector $\tilde{r}$, which were obtained by applying Eq. 16. Afterwards, the countries were ranked according Eq. 17 (Table A5b, Annex A). Since the distances were expressed in crisp numbers, no de-fuzziness was necessary.

Finally, the fuzzy Full Multiplicative Form was applied according to Eq. 18. Computation of fuzzy products $\tilde{A}_i$ and $\tilde{B}_i$ was a prerequisite for further calculations (Eq. 4, 5). Since $\tilde{U}_i^{'}$ is also a fuzzy number, Eq. 8 was applied to transform it into a crisp one (Annex B, Table B1). MULTIMOORA should summarize ranks from the Ratio System, Reference Point, and the Full Multiplicative Form.

6. Cardinal and ordinal scales in MULTIMOORA

Does there not exist a problem when MULTIMOORA has to totalize ranks from the Ratio System, Reference Point and the Full Multiplicative Form? Indeed adding up of ranks, ranks mean an ordinal scale (1^{st}, 2^{nd}, 3^{rd} etc.) signifies a return to a cardinal operation (1 + 2 + 3 + …). Is this allowed?

The answer is "no" following the Noble prize Winner Arrow:

6. 1. The impossibility theorem of arrow

"Obviously, a cardinal utility implies an ordinal preference but not *vice versa*" (Arrow 1974).

6. 2. The rank correlation method

The method of correlation of ranks consists of totalizing ranks. Rank correlation was introduced first by psychologists such as Spearman (1904, 1906 and 1910) and later taken over by the statistician Kendall in 1948. He argues (Kendall 1948): "we shall often operate with these numbers as if they were the cardinals of ordinary arithmetic, adding them, subtracting them and even multiplying them", but he never gives a proof of this statement. In his later work this statement is dropped (Kendall and Gibbons 1990).

In ordinal ranking 3 is farther away from 1 than 2 from 1, but Kendal (1948) goes too far (Table 2).

Table 2. Ordinal versus cardinal: comparing the price of one commodity

	Ordinal	Cardinal
	1	
	2	
	3	
	4	
A	5	6.03$
	6	6.02$
	7	6.01$
B	8	6$

For Kendal B is far away from A as it has 7 ranks before and A only 4, whereas it is not true cardinally.

In addition a supplemental notion, the statistical term of Correlation, is introduced. Suppose the statistical universe is just represented by two experts, for us it could be two methods. If they both rank in a same order different items to reach a certain goal, it is said that the correlation is perfect. However, perfect correlation is a rather exceptional situation. The problem is then posited: how in other situations correlation is measured. Therefore, the following Spearman's coefficient is used (Kendall 1948: 8):

$$\rho = 1 - \frac{6\sum D^2}{N(N^2 - 1)}, \tag{20}$$

where D stands for the difference between paired ranks, and N for the number of items ranked.

According to this formula, perfect correlation yields the coefficient of one. An acceptable correlation reaches the coefficient of one as much as possible. No correlation at all yields a coefficient of zero. If the series are exactly in reverse order, there will be a negative correlation of minus one, as shown in the following example (Table 3).

Table 3. Negative rank order correlations

Items	Expert 1	Expert 2	D	D^2
1	1	7	–6	36
2	2	6	–4	16
3	3	5	–2	4
4	4	4	0	0
5	5	3	2	4
6	6	2	4	16
7	7	1	6	36
Σ				112

This table shows that the sum of ranks in the case of an ordinal scale has no sense. Correlation leads to: $\rho = 1 - 6 \times 112 / (7(49 - 1)) = -1$. However, as addition of ranks is not allowed also a subtraction, the difference *D*, is not permitted.

Most people will better understand the ordinal problem by the way of a qualitative scale, e. g.:

1^{st} very good;
2^{nd} moderate;
3^{rd} very bad.

But equally one could say:

1^{st} very good;
2^{nd} good;
3^{rd} more or less good;
4^{th} moderate;
5^{th} more or less low;
6^{th} low;
7^{th} very low.

How is the first 2^{nd} comparable with the second 2^{nd}?, etc.

6.3. Arbitrary methods to go from an ordinal scale to a cardinal scale

1. Arithmetical Progression: 1, 2, 3, 4, 5, …

The ordinal scale 5 gets 1 cardinal point with all variations possible e.g. an additional point 1, etc.

The ordinal scale 4 gets 2 cardinal points etc.

The best one in the ordinal scale gets the most cardinal points in an arithmetical progression.

2. A Geometric Progression: 1, 2, 4, 8, 16, …

3. The Fundamental Scale of Saaty (1987): 1, 3, 5, 7, 9.

4. The Normal Scale of Lootsma (1987):

$e^{\circ} = 1$;
$e^1 = 2.7$;
$e^2 = 7.4$;
$e^3 = 20.1$ …

5. The Stretched Scale of Lootsma (1987):

$e^{\circ} = 1$;
$e^2 = 7.4$;
$e^4 = 54.6$;
$e^6 = 403.4$ …

6. The Point of View of the Psychologists (Miller 1956):

Ordinal Scales: 1, 2, 3, 4, 5, 6, 7.

After 7 an individual would no more know the cardinal significance compared to the previous 7 ones.

In fact infinite variations are possible. All stress an acceleration or a dis-acceleration process but are not aware of a possible trend break. The full multiplicative method with its huge numbers illustrates the best this trend break as shown in next Table 4.

Table 4. Ranking of Scenarios for the Belgian Regions by the Full-Multiplicative Method at the Year 1996

1	Scenario IX	Optimal Economic Policy in Wallonia and Brussels	203,267
2	Scenario X	Optimal Economic Policy in Wallonia and Brussels even agreeing on the Partition of the National Public Debt	196,306
3	Scenario VII	Flanders asks for the Partition of the National Public Debt	164,515
4	Scenario VIII	No Solidarity at all	158,881
5	Scenario II	Unfavorable Growth Rate for Flanders	90
6	Scenario IV	an Unfavorable Growth Rate for Flanders and at that moment asks also for the Partition of the National Public Debt	87
7	Scenario III	Partition of the National Public Debt	54
8	Scenario I	the Average Belgian	51
9	Scenario V	Average Belgian but as compensation Flanders asks for the Partition of the National Public Debt	49
10	Scenario O	Status Quo	43
11	Scenario VI	Flanders asks for the Partition of the National Public Debt	42

Source: Brauers, Ginevičius 2010.

With the usual Arithmetical Progression: 1, 2, 3, 4, 5, … the distance from the rank 4 to 5 would be the same as from 3 to 4 which is certainly not the case here. In addition all the other progressions fail to discover a trend break too.

Summarizing all these statements the following axioms are proposed.

6.4. Axioms on Ordinal and Cardinal Scales

1. *A deduction of an Ordinal Scale, a ranking, from cardinal data is always possible.*
2. *An Ordinal Scale can never produce a series of cardinal numbers.*
3. *An Ordinal Scale of a certain kind, a ranking, can be translated in an ordinal scale of another kind.*

In application of axiom 3 we shall translate the rankings of three methods of MULTIMOORA into an other ordinal scale based on *Dominance*, *being Dominated, Transitivity and Equability*.

6.5. Dominance, being Dominated, Transitiveness and Equability

The three methods of MULTIMOORA are assumed to have the same importance. Stakeholders, or their representatives like experts, may give a different importance in an ordinal ranking but this is not the case with the three methods of MULTIMOORA. These three methods represent all existing methods in multi-objective optimization with dimensionless measures and consequently all the three have the same important significance.

Dominance[3]

Absolute Dominance means that an alternative, solution or project dominates in ranking all other alternatives, solutions or projects which are all being dominated. This absolute dominance shows as rankings for MULTIMOORA: (1–1–1).

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3 Brauers and Zavadskas (2011) developed the theory of Dominance for the first time in January 2011.

General Dominance in two of the three methods is of the form with a < b < c <d:

(d–a–a) is generally dominating (c–b–b);

(a–d–a) is generally dominating (b–c–b);

(a–a–d) is generally dominating (b–b–c);

and further transitiveness plays fully.

Transitiveness. If *a* dominates *b* and *b* dominates *c* than also *a* will dominate *c*.

Overall Dominance of one alternative on another: (a–a–a) overall dominating (b–b–b), see Table 5.

Equability

Absolute Equability has the form**:** (e–e–e) for 2 alternatives.

Partial Equability of 2 on 3 exists e. g. (5–e–7) and (6–e–3).

A distinction can be made if a classification shows equability but one of the two alternatives belongs to a higher classified group.

Circular Reasoning

Despite all distinctions in classification some contradictions remain possible in a kind of Circular Reasoning. In such a case the same ranking is given.

Table 5. European Member States overall dominating other European Member States

Overall dominating	Overall being dominated
Germany (8–8–3)	France (10–10–4)
Ireland (9–11–5)	Spain (14–13–6)
Lithuania (21–21–25)	Malta (23–22–26)
Slovakia (26–26–16)	Bulgaria (27–27–23)

7. Application on the Multi-Objective Optimization of the European Union Member States based on MULTIMOORA

All Member States were assigned either of three roles in the European world–system. Best performing states with ranks from 1 to 9 were considered as *Core* states (Group 1), those possessing ranks 10–18 – as *Semi–Peripheral* states (Group 2), and those with ranks 19–27 – as *Peripheral States* (Group 3). It should be noted that all European states are unequivocally semi–peripheral at least in the *total* world–system, thus the given classification is only valid in the context of the *European* world–system (for the global world–system see for instance: Clark 2010).

Beside the general characteristics given above additional remarks have to be made for application on the European situation:

- We have to repeat again that with ranking by dominance the application remains in the ordinal sphere.
- We have to repeat again that the three methods have the same importance.
- Due to limited data availability and to limit the number of calculations only the years 2000, 2004 and 2008 were selected. In that way the response matrix was already composed of 972 elements.

– Also the choice of the years 2000, 2004 and 2008 has an historical meaning. In 2000 the European Union was only composed of 15 countries, the so–called EU–15: the original countries (1957) BENELUX (Belgium, Netherlands, Luxemburg), France, Germany and Italy; UK, Ireland and Denmark (1973); Greece (1981); Spain and Portugal (1986).

On May 1, 2004 the EU extended with 10 members: Poland, Lithuania, Latvia, Estonia, Slovenia, Slovakia, Czech Republic, Hungary, Cyprus and Malta. Consequently these countries were not member in 2000, a half time in 2004 and full time in 2008. Nevertheless their data are also assembled for 2000 and 2004.

On January 1, 2007 Romania and Bulgaria joined the Union meaning that they were not present in 2000 and 2004. Nevertheless their data are also used for 2000 and 2004.

– No *Equability* in ranking was found between the EU members.
– No *Absolute Dominance* was present in the three methods.
– *General Dominance*: Sweden with (1–5–7) dominates Luxemburg (2–2–19) and further all the others by transitiveness.

Table 6 and Annex D show the final results for the European Member States on basis of Dominance.

Table 6. MOO Ranking on basis of 12 Structural Indicators for the 27 Member States of the EU

Ranking	Member States with MULTIMOORA Rankings
	Core (Group 1)
1	Sweden (1–5–7)
2	Luxemburg (2–2–19)
3	Finland (4–9–1)
4	Austria (5–3–9)
5	Netherlands (6–1–14)
6	Denmark (3–4–18)
7	Belgium (11–6–2)
8	UK (7–7–15)
9	Germany (8–8–3)
	Semi-Periphery (Group 2)
10	France (10–10–4)
11	Ireland (9–11–5)
12	Spain (14–13–6)
13	Italy (16–12–8)
14	Slovenia (12–14–12)
15	Portugal (17–15–11)
16	Czech (13–16–17)
17	Greece (22–17–10)
18	Estonia (19–19–13)
	Periphery (Group 3)
19	Cyprus (15–24–20)
20	Hungary (18–18–21)

End of Table 6

Periphery (Group 3)	
21	Poland (24–20–22)
22	Lithuania (21–21–25)
23	Malta (23–22–26)
24	Latvia (20–23–27)
25	Romania (25–25–24)
26	Slovakia (26–26–16)
27	Bulgaria (27–27–23)

For details see: Annex C.

The application of a theory of Dominance to solve the ordinal problem was successful. If the transition from cardinal to ordinal is possible but from ordinal to cardinal not then the solution has to be found in the transition from one ordinal system to another one. Let us hope that in this way the old discussion between cardinal and ordinal is solved once for all.

Given the recession of 2009 a trend break occurred which was certainly fatal for Ireland, Greece, Portugal and even perhaps for the UK. *Standard&Poor's* gives a credit rating of BB+ to Greece, which means classifying its government bonds as "junk" paper. Before March 2009 Ireland had the highest rating of AAA but since then it went down over AA+ , AA, AA–, A+ to A. Portugal has even A –. Of course this is only a single indicator. Bur the rating offices take into account many criteria[4]. Probably Ireland, Portugal and Greece will have to substitute Group 2 (Semi Periphery) by Group 3 (Periphery). One can even wonder if UK can stay in Group 1. Consequently similar research on the year 2009 would be very useful.

8. Conclusion

Fuzzy logic handles vague problems in various areas. Fuzzy numbers can represent either quantitative or qualitative variables. The quantitative fuzzy variables can embody crisp numbers, aggregates of historical data (i.e. time series) or forecasts. The qualitative fuzzy variables may be applied when dealing with ordinal scales. The MULTIMOORA method was therefore updated with fuzzy number theory. Vertex method was used when measuring the distances between fuzzy numbers. Centre of area method was applied for defuzzification.

The MULTIMOORA method consists of three parts, namely Ratio System, Reference Point and Full Multiplicative Form. Accordingly, each of them was modified and thus updated with triangular fuzzy number theory. The fuzzy Ratio System defines internal normalization, aggregation of criteria into single ratios and defuzzification. The fuzzy Reference Point approach relies on definition of the Maximal Objective Reference Point as well as measurement of distances between certain coordinates of the Reference Point and every alternative according to vertex method. The fuzzy Full Multiplicative Form embodies maximization of a purely multiplicative utility function and defuzzification. The fuzzy MULTIMOORA summarizes these three approaches under the form of three sets of ranking, which means: of an ordinal

[4] Vertrouwen in Ierland slinkt met de dag, *De Tijd*, November 25, 2010. These figures are considered as confidential, but the newspaper takes the responsibility of publication.

order. At that moment the problem is set: what to do with these three sets of rankings. With small responses matrices no problems did arrive. The solution was mostly easy to see. For large matrices it is much more complicated.

At that occasion three Axioms on Ordinal and Cardinal Scales are proposed:

1. *A deduction of an Ordinal Scale, a ranking, from cardinal data is always possible.*
2. *An Ordinal Scale can never produce a series of cardinal numbers.*
3. *An Ordinal Scale of a certain kind, a ranking, can be translated in an ordinal scale of another kind.*

In application of axiom 3 the rankings of the three methods of MULTIMOORA were translated into an other ordinal scale based on *Dominance*, *being Dominated, Transitivity and Equability*.

The three methods of MULTIMOORA are assumed to have the same importance. These three methods represent all existing methods with dimensionless measures in multi-objective optimization and all the three have an important significance.

Fuzzy MULTIMOORA ranked the EU Member States in three groups based on the cited domination principles and according to their performance in reaching the goals of the Lisbon Strategy 2000–2008. As table 6 suggests, the best performing countries (Group 1) are Sweden, Luxemburg Finland, Austria, the Netherlands Denmark Belgium, UK and Germany. Group 2 consists of, France, Ireland, Spain, Italy, Slovenia, Portugal, Czech Republic, Greece, and Estonia. Group 3 encompasses the less performing states, namely Cyprus, Hungary, Poland, Lithuania, Malta, Latvia, Romania, Slovakia and Bulgaria. The three groups are called successively: Core, Semi-Periphery and Periphery in comparison with what is done on world level.

Given the recession of 2009 a trend break occurred which was certainly fatal for Ireland, Greece, Portugal and even perhaps for the UK. Consequently new research on 2010 would be very useful. Nevertheless no link has to be made with the period from before 2010. The changes are too profound.

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Annex A. The fuzzy MOORA method

Table A1. The initial data for the research of EU Member States performance in seeking Lisbon goals (2000–2008)

	1. GDP per capita in PPS			**2. Labor productivity per person employed**			**3. Employment rate by gender; Total**			**4. Employment rate of older workers by gender; Total**			**5. Youth education attainment level**			**6. Gross domestic expenditure on R&D (GERD)**			**7. Business investment**			**8. Comparative price levels**			**9. At-risk-of-poverty rate after social transfers**			**10. Long-term unemployment rate**			**11. Greenhouse gas emissions**			**12. Energy intensity of the economy**		
	2000	2004	2008	2000	2004	2008	2000	2004	2008	2000	2004	2008	2000	2004	2008	2000	2004	2008	2000	2004	2008	2000	2004	2008	2000	2004	2008	2000	2004	2008	2000	2004	2008	2000	2004	2008
BE	126	121	115	136.6	131.7	125.4	60.5	60.3	62.4	26.3	30	34.5	81.7	81.8	82.2	1.97	1.86	1.92	19.1	18.2	21	102	106.7	111.1	13	14.3	14.7	3.7	4.1	3.3	100.9	101.3	92.9	243.7	229.3	199.8
BG	28	34	41	30.4	33.7	37.2	50.4	54.2	64	20.8	32.5	46	75.2	76.1	83.7	0.52	0.5	0.49	12.1	17.6	27.7	38.7	42	50.2	14	15	21.4	9.4	7.2	2.9	59	60.6	62.6	1362.4	1139.3	944.2
CZ	68	75	80	61.8	68	71.9	65	64.2	66.6	36.3	42.7	47.6	91.2	91.4	91.6	1.21	1.25	1.47	24.4	21	19	48.1	55.4	72.8	8	10.4	9	4.2	4.2	2.2	75.6	74.8	72.5	659.1	660.2	525.3
DK	132	126	120	110.5	108.6	101	76.3	75.7	78.1	55.7	60.3	57	72	76.2	71	2.24	2.48	2.72	18.5	17.4	19	130.2	139.5	141.2	10	10.9	11.8	0.9	1.2	0.5	99.1	98.7	92.6	112.5	111.9	103.1
DE	118	116	116	108	108.1	106.9	65.6	65	70.7	37.6	41.8	53.8	74.7	72.8	74.1	2.45	2.49	2.63	19.7	16.1	17.5	106.5	104.7	103.8	10	12.2	15.2	3.8	5.5	3.8	83.2	81.2	77.8	166.0	166.1	151.1
EE	45	57	67	46.9	57.4	63.8	60.4	63	69.8	46.3	52.4	62.4	79	80.3	82.2	0.6	0.85	1.29	22	27.1	24	57.2	63	78	18	20.2	19.5	6.3	5	1.7	44.5	49.3	49.6	812.7	687.5	570.5
IE	131	142	135	127.4	135.2	130.1	65.2	66.3	67.6	45.3	49.5	53.7	82.6	85.3	87.7	1.12	1.23	1.43	19.6	20.9	16.6	114.8	125.9	127.6	20	20.7	15.4	1.6	1.6	1.7	123.6	122.8	123	137.0	123.0	106.5
EL	84	94	94	93.6	101.1	102.1	56.5	59.4	61.9	39	39.4	42.8	79.2	83	82.1	0.59	0.55	0.58	17.9	18.7	16.6	84.8	87.6	94	20	19.9	20.1	6.2	5.6	3.6	120.9	125.7	122.8	204.6	186.8	170.0
ES	97	101	103	103.7	102	103.6	56.3	61.1	64.3	37	41.3	45.6	66	61.2	60	0.91	1.06	1.35	22.7	24.7	25	85	91	95.4	18	19.9	19.6	4.6	3.4	2	133.6	147.5	142.3	196.2	198.1	176.4
FR	115	110	108	125	120.6	121.2	62.1	63.8	64.9	29.9	37.8	38.2	81.6	81.8	83.4	2.15	2.15	2.02	16.4	16.2	18.5	105.8	109.9	110.8	16	13.5	13.4	3.5	3.8	2.9	98.9	98.1	93.6	179.1	179.4	166.7
IT	117	107	102	126	112.1	109.4	53.7	57.6	58.7	27.7	30.5	34.4	69.4	73.4	76.5	1.05	1.1	1.18	18	18.1	18.5	97.5	104.9	105.6	18	19.1	18.7	6.3	4	3.1	106.3	111	104.7	146.6	150.5	142.6
CY	89	90	96	85	82.8	87.2	65.7	68.9	70.9	49.4	49.9	54.8	79	77.6	85.1	0.24	0.37	0.46	11.9	12.1	20.4	88	91.2	90.5	15	15	16.2	1.2	1.2	0.5	172.8	176.4	193.9	237.1	215.5	213.4
LV	37	46	57	40.2	45.7	52	57.5	62.3	68.6	36	47.9	59.4	76.5	79.5	80	0.44	0.42	0.61	22.9	24.4	24.5	58.8	56.1	72.6	16	19.2	25.6	7.9	4.6	1.9	38.1	41.1	44.4	441.0	387.0	308.7
LT	39	50	62	42.7	53.3	62	59.1	61.2	64.3	40.4	47.1	53.1	78.9	85	89.1	0.59	0.75	0.8	16.4	18.8	20.2	52.6	53.5	64.7	17	20.7	20	8	5.8	1.2	39	44.2	48.9	571.2	547.4	417.5
LU	244	253	276	175.9	169.6	175.7	62.7	62.5	63.4	26.7	30.4	34.1	77.5	72.5	72.8	1.65	1.63	1.62	17	17.3	16.1	101.5	103	119.1	12	12.7	13.4	0.5	1	1.6	75.5	100.7	95.2	165.3	185.6	154.6
HU	55	63	64	57.7	67.4	71.2	56.3	56.8	56.7	22.2	31.1	31.4	83.5	83.5	83.6	0.79	0.87	1	20.2	19	18	49.2	62	68.1	11	13.5	12.4	3.1	2.7	3.6	79.2	81.2	75.1	487.5	435.3	401.4
MT	84	77	76	96.7	90	86.9	54.2	54	55.3	28.5	31.5	29.2	40.9	51	53	0.26	0.53	0.54	13.3	10.2	11	73.2	73.2	78.8	15	13.7	14.6	4.5	3.4	2.5	126.9	140.6	144.2	191.3	217.4	194.9
NL	134	129	134	114.4	112.2	114.4	72.9	73.1	77.2	38.2	45.2	53	71.9	75	76.2	1.82	1.81	1.63	18.8	15.6	16.9	100	106.1	104	11	10.7	10.5	0.8	1.6	1	101.2	102.9	97.6	184.8	191.6	171.6
AT	131	127	124	120.6	117.5	114	68.5	67.8	72.1	28.8	28.8	41	85.1	85.8	84.5	1.94	2.26	2.67	22.5	20.8	21	101.8	103.3	105.1	12	12.8	12.4	1	1.4	0.9	102.7	116.3	110.8	140.3	151.7	138.1
PL	48	51	56	55.2	61.5	62	55	51.7	59.2	28.4	26.2	31.6	88.8	90.9	91.3	0.64	0.56	0.61	21.4	14.7	17.5	57.9	53.2	69.1	16	20.5	16.9	7.4	10.3	2.4	86.1	85.3	87.3	488.7	442.1	383.5
PT	81	77	78	71.5	69.3	73.5	68.4	67.8	68.2	50.7	50.3	50.8	43.2	49.6	54.3	0.76	0.77	1.51	24.1	20.3	20	83	87.4	87	21	20.4	18.5	1.7	3	3.7	137.1	142.8	132.2	197.5	201.3	181.5
RO	26	34	42	23.6	34.4	50.2	63	57.7	59	49.5	36.9	43.1	76.1	75.3	78.3	0.37	0.39	0.58	15.4	18.7	26.4	42.5	43.3	60.9	17	18	23.4	3.8	4.8	2.4	56.3	64.2	60.3	913.4	768.3	614.6
SI	80	86	91	76.2	82	84.3	62.8	65.3	68.6	22.7	29	32.8	88	90.5	90.2	1.39	1.4	1.66	22.4	21.5	24.6	72.8	75.5	82.3	11	12.2	12.3	4.1	3.2	1.9	101.9	107.7	115.2	299.2	289.6	257.5
SK	50	57	72	58	65.4	79.2	56.8	57	62.3	21.3	26.8	39.2	94.8	91.7	92.3	0.65	0.51	0.47	23.8	22.2	23	44.4	54.9	70.2	13.3	13.3	10.9	10.3	11.8	6.6	66.6	68.7	66.1	796.4	729.1	519.7
FI	117	116	117	114.8	112.9	111.8	67.2	67.6	71.1	41.6	50.9	56.5	87.7	84.5	86.2	3.35	3.45	3.73	17.6	16.5	19	120.8	123.8	124.3	11	11	13.6	2.8	2.1	1.2	98.2	114	99.7	246.3	257.4	217.8
SE	128	126	122	114.3	114.9	112.3	73	72.1	74.3	64.9	69.1	70.1	85.2	86	85.6	3.61	3.62	3.75	15.2	14.1	16.8	127.6	121.4	114.5	8	11.3	12.1	1.4	1.5	0.8	95.1	97.2	88.3	177.4	177.5	152.1
UK	119	124	116	110.7	113.8	109.7	71.2	71.7	71.5	50.7	56.2	58	76.7	77	78.2	1.81	1.68	1.88	15.9	14.9	14.4	119.9	108.5	100.1	19	18	18.8	1.4	1	1.4	87.2	85.4	81.4	144.5	131.0	113.7

Table A2. Fuzzy response matrix $\tilde{X}$

j	1			2			3			4			5			6		
	min	**average**	**max**	**min**	**average**	**max**	**min**	**average**	**max**	**min**	**average**	**max**	**min**	**average**	**max**	**min**	**average**	**max**
BE	115.00	120.58	126.00	125.40	131.15	136.60	60.30	61.06	62.40	26.30	30.08	34.50	81.70	81.90	82.20	1.86	1.92	1.97
BG	28.00	33.92	41.00	30.40	33.65	37.20	50.40	55.92	64.00	20.80	31.45	46.00	75.20	78.24	83.70	0.49	0.50	0.52
CZ	68.00	74.17	80.00	61.80	67.10	71.90	64.20	65.26	66.60	36.30	41.94	47.60	91.20	91.40	91.60	1.21	1.31	1.47
DK	120.00	125.90	132.00	101.00	106.62	110.50	75.70	76.69	78.10	55.70	57.63	60.30	71.00	73.03	76.20	2.24	2.47	2.72
DE	116.00	116.66	118.00	106.90	107.67	108.10	65.00	67.05	70.70	37.60	43.89	53.80	72.80	73.86	74.70	2.45	2.52	2.63
EE	45.00	55.60	67.00	46.90	55.59	63.80	60.40	64.28	69.80	46.30	53.30	62.40	79.00	80.49	82.20	0.60	0.87	1.29
IE	131.00	135.92	142.00	127.40	130.86	135.20	65.20	66.36	67.60	45.30	49.38	53.70	82.60	85.17	87.70	1.12	1.25	1.43
EL	84.00	90.54	94.00	93.60	98.86	102.10	56.50	59.23	61.90	39.00	40.36	42.80	79.20	81.42	83.00	0.55	0.57	0.59
ES	97.00	100.30	103.00	102.00	103.10	103.70	56.30	60.48	64.30	37.00	41.15	45.60	60.00	62.35	66.00	0.91	1.09	1.35
FR	108.00	110.96	115.00	120.60	122.25	125.00	62.10	63.59	64.90	29.90	35.08	38.20	81.60	82.26	83.40	2.02	2.11	2.15
IT	102.00	108.49	117.00	109.40	115.61	126.00	53.70	56.63	58.70	27.70	30.75	34.40	69.40	73.04	76.50	1.05	1.11	1.18
CY	89.00	91.62	96.00	82.80	84.98	87.20	65.70	68.47	70.90	49.40	51.31	54.80	77.60	80.50	85.10	0.24	0.34	0.46
LV	37.00	45.95	57.00	40.20	45.71	52.00	57.50	62.64	68.60	36.00	46.79	59.40	76.50	78.65	80.00	0.42	0.48	0.61
LT	39.00	49.45	62.00	42.70	52.06	62.00	59.10	61.50	64.30	40.40	46.58	53.10	78.90	84.23	89.10	0.59	0.71	0.80
LU	244.00	257.32	276.00	169.60	173.71	175.90	62.50	62.87	63.40	26.70	30.25	34.10	72.50	74.23	77.50	1.62	1.63	1.65
HU	55.00	60.53	64.00	57.70	65.18	71.20	56.30	56.60	56.80	22.20	27.88	31.40	83.50	83.53	83.60	0.79	0.88	1.00
MT	76.00	78.92	84.00	86.90	91.11	96.70	54.00	54.50	55.30	28.50	29.71	31.50	40.90	47.99	53.00	0.26	0.42	0.54
NL	129.00	132.31	134.00	112.20	113.66	114.40	72.90	74.37	77.20	38.20	45.06	53.00	71.90	74.34	76.20	1.63	1.75	1.82
AT	124.00	127.30	131.00	114.00	117.34	120.60	67.80	69.44	72.10	28.80	32.40	41.00	84.50	85.13	85.80	1.94	2.27	2.67
PL	48.00	51.56	56.00	55.20	59.48	62.00	51.70	55.22	59.20	26.20	28.65	31.60	88.80	90.33	91.30	0.56	0.60	0.64
PT	77.00	78.65	81.00	69.30	71.41	73.50	67.80	68.13	68.40	50.30	50.60	50.80	43.20	48.82	54.30	0.76	0.96	1.51
RO	26.00	33.36	42.00	23.60	34.41	50.20	57.70	59.86	63.00	36.90	42.86	49.50	75.30	76.56	78.30	0.37	0.44	0.58
SI	80.00	85.55	91.00	76.20	80.76	84.30	62.80	65.52	68.60	22.70	27.85	32.80	88.00	89.56	90.50	1.39	1.48	1.66
SK	50.00	58.98	72.00	58.00	66.97	79.20	56.80	58.65	62.30	21.30	28.18	39.20	91.70	92.92	94.80	0.47	0.54	0.65
FI	116.00	116.67	117.00	111.80	113.16	114.80	67.20	68.61	71.10	41.60	49.27	56.50	84.50	86.12	87.70	3.35	3.51	3.73
SE	122.00	125.31	128.00	112.30	113.83	114.90	72.10	73.13	74.30	64.90	68.00	70.10	85.20	85.60	86.00	3.61	3.66	3.75
UK	116.00	119.62	124.00	109.70	111.39	113.80	71.20	71.47	71.70	50.70	54.88	58.00	76.70	77.30	78.20	1.68	1.79	1.88
Sum	274978	301706	335446	236452	255155	275759	104826	111483	120374	39441	49176	62528	161567	169429	178389	65	73	86
sq	524	549	579	486	505	525	324	334	347	199	222	250	402	412	422	8	9	9

TableA2 continued

j	7			8			9			10			11			12		
	min	average	max	min	average	max	min	average	max	min	average	max	min	average	max	min	average	max
BE	18.20	19.40	21.00	102.00	106.54	111.10	13.00	13.98	14.70	3.30	3.69	4.10	92.90	98.29	101.30	199.82	223.51	243.68
BG	12.10	18.07	27.70	38.70	43.37	50.20	14.00	16.50	21.40	2.90	5.81	9.40	59.00	60.72	62.60	944.16	1135.85	1362.36
CZ	19.00	21.35	24.40	48.10	57.89	72.80	8.00	9.08	10.40	2.20	3.39	4.20	72.50	74.29	75.60	525.30	611.44	660.22
DK	17.40	18.29	19.00	130.20	136.88	141.20	10.00	10.88	11.80	0.50	0.81	1.20	92.60	96.75	99.10	103.13	109.07	112.47
DE	16.10	17.71	19.70	103.80	104.99	106.50	10.00	12.29	15.20	3.80	4.30	5.50	77.80	80.70	83.20	151.12	160.92	166.12
EE	22.00	24.28	27.10	57.20	65.51	78.00	18.00	19.21	20.20	1.70	3.77	6.30	44.50	47.74	49.60	570.51	683.12	812.71
IE	16.60	18.95	20.90	114.80	122.63	127.60	15.40	18.54	20.70	1.60	1.63	1.70	122.80	123.13	123.60	106.52	121.52	137.00
EL	16.60	17.71	18.70	84.80	88.72	94.00	19.90	20.00	20.10	3.60	5.00	6.20	120.90	123.12	125.70	169.95	186.56	204.57
ES	22.70	24.11	25.00	85.00	90.37	95.40	18.00	19.15	19.90	2.00	3.15	4.60	133.60	141.02	147.50	176.44	189.97	198.07
FR	16.20	17.00	18.50	105.80	108.81	110.80	13.40	14.25	16.00	2.90	3.38	3.80	93.60	96.84	98.90	166.74	174.97	179.36
IT	18.00	18.20	18.50	97.50	102.60	105.60	18.00	18.59	19.10	3.10	4.27	6.30	104.70	107.30	111.00	142.59	146.54	150.53
CY	11.90	14.32	20.40	88.00	89.89	91.20	15.00	15.39	16.20	0.50	0.90	1.20	172.80	180.80	193.90	213.39	221.72	237.06
LV	22.90	23.92	24.50	56.10	62.10	72.60	16.00	19.89	25.60	1.90	4.10	7.90	38.10	41.12	44.40	308.74	374.91	441.00
LT	16.40	18.40	20.20	52.60	56.68	64.70	17.00	19.16	20.70	1.20	3.82	8.00	39.00	43.85	48.90	417.54	507.30	571.22
LU	16.10	16.79	17.30	101.50	107.58	119.10	12.00	12.69	13.40	0.50	0.93	1.60	75.50	89.79	100.70	154.61	168.04	185.63
HU	18.00	19.05	20.20	49.20	59.22	68.10	11.00	12.26	13.50	2.70	3.11	3.60	75.10	78.46	81.20	401.35	439.99	487.54
MT	10.20	11.43	13.30	73.20	75.02	78.80	13.70	14.42	15.00	2.50	3.37	4.50	126.90	137.03	144.20	191.27	200.85	217.38
NL	15.60	17.05	18.80	100.00	103.34	106.10	10.50	10.73	11.00	0.80	1.09	1.60	97.60	100.54	102.90	171.58	182.46	191.56
AT	20.80	21.42	22.50	101.80	103.39	105.10	12.00	12.40	12.80	0.90	1.08	1.40	102.70	109.79	116.30	138.06	143.24	151.71
PL	14.70	17.66	21.40	53.20	59.71	69.10	16.00	17.70	20.50	2.40	5.68	10.30	85.30	86.23	87.30	383.54	435.97	488.67
PT	20.00	21.39	24.10	83.00	85.78	87.40	18.50	19.94	21.00	1.70	2.66	3.70	132.20	137.30	142.80	181.53	193.22	201.25
RO	15.40	19.66	26.40	42.50	48.21	60.90	17.00	19.27	23.40	2.40	3.52	4.80	56.30	60.18	64.20	614.57	755.52	913.36
SI	21.50	22.80	24.60	72.80	76.76	82.30	11.00	11.82	12.30	1.90	2.92	4.10	101.90	108.13	115.20	257.54	281.52	299.15
SK	22.20	22.99	23.80	44.40	55.52	70.20	10.90	12.45	13.30	6.60	9.29	11.80	66.10	67.12	68.70	519.68	670.74	796.44
FI	16.50	17.67	19.00	120.80	122.96	124.30	11.00	11.81	13.60	1.20	1.92	2.80	98.20	103.73	114.00	217.79	239.91	257.39
SE	14.10	15.33	16.80	114.50	121.05	127.60	8.00	10.30	12.10	0.80	1.19	1.50	88.30	93.45	97.20	152.08	168.55	177.45
UK	14.40	15.05	15.90	100.10	109.20	119.90	18.00	18.59	19.00	1.00	1.25	1.40	81.40	84.63	87.20	113.66	129.10	144.54
Sum	8322	9889	12355	202131	225220	254215	5527	6604	8057	164	369	790	248335	272337	297810	3248647	4549209	6118770
sq	91	99	111	450	475	504	74	81	90	13	19	28	498	522	546	1802	2133	2474

Table A3. Normalized fuzzy response matrix $\tilde{X}^*$ (objectives divided by their square roots)

j	1			2			3			4			5			6		
	x^*_{ij1}	x^*_{ij2}	x^*_{ij3}	x^*_{ij1}	x^*_{ij2}	x^*_{ij3}	x^*_{ij1}	x^*_{ij2}	x^*_{ij3}	x^*_{ij1}	x^*_{ij2}	x^*_{ij3}	x^*_{ij1}	x^*_{ij2}	$a_1 \succ a_3$	x^*_{ij1}	x^*_{ij2}	x^*_{ij3}
BE	0.219	0.220	0.218	0.258	0.260	0.260	0.186	0.183	0.180	0.132	0.136	0.138	0.203	0.199	0.195	0.231	0.224	0.213
BG	0.053	0.062	0.071	0.063	0.067	0.071	0.156	0.167	0.184	0.105	0.142	0.184	0.187	0.190	0.198	0.061	0.059	0.056
CZ	0.130	0.135	0.138	0.127	0.133	0.137	0.198	0.195	0.192	0.183	0.189	0.190	0.227	0.222	0.217	0.150	0.153	0.159
DK	0.229	0.229	0.228	0.208	0.211	0.210	0.234	0.230	0.225	0.280	0.260	0.241	0.177	0.177	0.180	0.279	0.289	0.293
DE	0.221	0.212	0.204	0.220	0.213	0.206	0.201	0.201	0.204	0.189	0.198	0.215	0.181	0.179	0.177	0.305	0.295	0.284
EE	0.086	0.101	0.116	0.096	0.110	0.121	0.187	0.193	0.201	0.233	0.240	0.250	0.197	0.196	0.195	0.075	0.102	0.139
IE	0.250	0.247	0.245	0.262	0.259	0.257	0.201	0.199	0.195	0.228	0.223	0.215	0.205	0.207	0.208	0.139	0.147	0.154
EL	0.160	0.165	0.162	0.192	0.196	0.194	0.175	0.177	0.178	0.196	0.182	0.171	0.197	0.198	0.197	0.068	0.067	0.064
ES	0.185	0.183	0.178	0.210	0.204	0.197	0.174	0.181	0.185	0.186	0.186	0.182	0.149	0.151	0.156	0.113	0.128	0.146
FR	0.206	0.202	0.199	0.248	0.242	0.238	0.192	0.190	0.187	0.151	0.158	0.153	0.203	0.200	0.197	0.251	0.246	0.232
IT	0.195	0.198	0.202	0.225	0.229	0.240	0.166	0.170	0.169	0.139	0.139	0.138	0.173	0.177	0.181	0.131	0.130	0.127
CY	0.170	0.167	0.166	0.170	0.168	0.166	0.203	0.205	0.204	0.249	0.231	0.219	0.193	0.196	0.201	0.030	0.040	0.050
LV	0.071	0.084	0.098	0.083	0.090	0.099	0.178	0.188	0.198	0.181	0.211	0.238	0.190	0.191	0.189	0.052	0.056	0.066
LT	0.074	0.090	0.107	0.088	0.103	0.118	0.183	0.184	0.185	0.203	0.210	0.212	0.196	0.205	0.211	0.073	0.083	0.086
LU	0.465	0.468	0.477	0.349	0.344	0.335	0.193	0.188	0.183	0.134	0.136	0.136	0.180	0.180	0.183	0.201	0.191	0.178
HU	0.105	0.110	0.111	0.119	0.129	0.136	0.174	0.170	0.164	0.112	0.126	0.126	0.208	0.203	0.198	0.098	0.103	0.108
MT	0.145	0.144	0.145	0.179	0.180	0.184	0.167	0.163	0.159	0.144	0.134	0.126	0.102	0.117	0.125	0.032	0.049	0.058
NL	0.246	0.241	0.231	0.231	0.225	0.218	0.225	0.223	0.223	0.192	0.203	0.212	0.179	0.181	0.180	0.203	0.205	0.196
AT	0.236	0.232	0.226	0.234	0.232	0.230	0.209	0.208	0.208	0.145	0.146	0.164	0.210	0.207	0.203	0.241	0.265	0.288
PL	0.092	0.094	0.097	0.114	0.118	0.118	0.160	0.165	0.171	0.132	0.129	0.126	0.221	0.219	0.216	0.070	0.070	0.069
PT	0.147	0.143	0.140	0.143	0.141	0.140	0.209	0.204	0.197	0.253	0.228	0.203	0.107	0.119	0.129	0.094	0.112	0.163
RO	0.050	0.061	0.073	0.049	0.068	0.096	0.178	0.179	0.182	0.186	0.193	0.198	0.187	0.186	0.185	0.046	0.051	0.063
SI	0.153	0.156	0.157	0.157	0.160	0.161	0.194	0.196	0.198	0.114	0.126	0.131	0.219	0.218	0.214	0.173	0.173	0.179
SK	0.095	0.107	0.124	0.119	0.133	0.151	0.175	0.176	0.180	0.107	0.127	0.157	0.228	0.226	0.224	0.058	0.063	0.070
FI	0.221	0.212	0.202	0.230	0.224	0.219	0.208	0.205	0.205	0.209	0.222	0.226	0.210	0.209	0.208	0.417	0.410	0.402
SE	0.233	0.228	0.221	0.231	0.225	0.219	0.223	0.219	0.214	0.327	0.307	0.280	0.212	0.208	0.204	0.449	0.428	0.405
UK	0.221	0.218	0.214	0.226	0.221	0.217	0.220	0.214	0.207	0.255	0.247	0.232	0.191	0.188	0.185	0.209	0.209	0.203

Table A3 continued

j	7			8			9			10			11			12		
	x^*_{ij1}	x^*_{ij2}	x^*_{ij3}	x^*_{ij1}	x^*_{ij2}	x^*_{ij3}	x^*_{ij1}	x^*_{ij2}	x^*_{ij3}	x^*_{ij1}	x^*_{ij2}	x^*_{ij3}	x^*_{ij1}	x^*_{ij2}	x^*_{ij3}	x^*_{ij1}	x^*_{ij2}	x^*_{ij3}
BE	0.200	0.195	0.189	0.227	0.224	0.220	0.175	0.172	0.164	0.257	0.192	0.146	0.186	0.188	0.186	0.111	0.105	0.099
BG	0.133	0.182	0.249	0.086	0.091	0.100	0.188	0.203	0.238	0.226	0.302	0.334	0.118	0.116	0.115	0.524	0.533	0.551
CZ	0.208	0.215	0.220	0.107	0.122	0.144	0.108	0.112	0.116	0.172	0.176	0.149	0.145	0.142	0.139	0.291	0.287	0.267
DK	0.191	0.184	0.171	0.290	0.288	0.280	0.135	0.134	0.131	0.039	0.042	0.043	0.186	0.185	0.182	0.057	0.051	0.045
DE	0.176	0.178	0.177	0.231	0.221	0.211	0.135	0.151	0.169	0.296	0.224	0.196	0.156	0.155	0.152	0.084	0.075	0.067
EE	0.241	0.244	0.244	0.127	0.138	0.155	0.242	0.236	0.225	0.133	0.196	0.224	0.089	0.091	0.091	0.317	0.320	0.329
IE	0.182	0.191	0.188	0.255	0.258	0.253	0.207	0.228	0.231	0.125	0.085	0.060	0.246	0.236	0.226	0.059	0.057	0.055
EL	0.182	0.178	0.168	0.189	0.187	0.186	0.268	0.246	0.224	0.281	0.260	0.221	0.243	0.236	0.230	0.094	0.087	0.083
ES	0.249	0.242	0.225	0.189	0.190	0.189	0.242	0.236	0.222	0.156	0.164	0.164	0.268	0.270	0.270	0.098	0.089	0.080
FR	0.178	0.171	0.166	0.235	0.229	0.220	0.180	0.175	0.178	0.226	0.176	0.135	0.188	0.186	0.181	0.093	0.082	0.073
IT	0.197	0.183	0.166	0.217	0.216	0.209	0.242	0.229	0.213	0.242	0.222	0.224	0.210	0.206	0.203	0.079	0.069	0.061
CY	0.130	0.144	0.184	0.196	0.189	0.181	0.202	0.189	0.180	0.039	0.047	0.043	0.347	0.346	0.355	0.118	0.104	0.096
LV	0.251	0.241	0.220	0.125	0.131	0.144	0.215	0.245	0.285	0.148	0.214	0.281	0.076	0.079	0.081	0.171	0.176	0.178
LT	0.180	0.185	0.182	0.117	0.119	0.128	0.229	0.236	0.231	0.094	0.199	0.285	0.078	0.084	0.090	0.232	0.238	0.231
LU	0.176	0.169	0.156	0.226	0.227	0.236	0.161	0.156	0.149	0.039	0.048	0.057	0.152	0.172	0.185	0.086	0.079	0.075
HU	0.197	0.192	0.182	0.109	0.125	0.135	0.148	0.151	0.150	0.211	0.162	0.128	0.151	0.150	0.149	0.223	0.206	0.197
MT	0.112	0.115	0.120	0.163	0.158	0.156	0.184	0.177	0.167	0.195	0.175	0.160	0.255	0.263	0.264	0.106	0.094	0.088
NL	0.171	0.171	0.169	0.222	0.218	0.210	0.141	0.132	0.123	0.062	0.057	0.057	0.196	0.193	0.189	0.095	0.086	0.077
AT	0.228	0.215	0.202	0.226	0.218	0.208	0.161	0.153	0.143	0.070	0.056	0.050	0.206	0.210	0.213	0.077	0.067	0.061
PL	0.161	0.178	0.193	0.118	0.126	0.137	0.215	0.218	0.228	0.187	0.295	0.366	0.171	0.165	0.160	0.213	0.204	0.198
PT	0.219	0.215	0.217	0.185	0.181	0.173	0.249	0.245	0.234	0.133	0.139	0.132	0.265	0.263	0.262	0.101	0.091	0.081
RO	0.169	0.198	0.238	0.095	0.102	0.121	0.229	0.237	0.261	0.187	0.183	0.171	0.113	0.115	0.118	0.341	0.354	0.369
SI	0.236	0.229	0.221	0.162	0.162	0.163	0.148	0.145	0.137	0.148	0.152	0.146	0.204	0.207	0.211	0.143	0.132	0.121
SK	0.243	0.231	0.214	0.099	0.117	0.139	0.147	0.153	0.148	0.515	0.484	0.420	0.133	0.129	0.126	0.288	0.314	0.322
FI	0.181	0.178	0.171	0.269	0.259	0.247	0.148	0.145	0.152	0.094	0.100	0.100	0.197	0.199	0.209	0.121	0.112	0.104
SE	0.155	0.154	0.151	0.255	0.255	0.253	0.108	0.127	0.135	0.062	0.062	0.053	0.177	0.179	0.178	0.084	0.079	0.072
UK	0.158	0.151	0.143	0.223	0.230	0.238	0.242	0.229	0.212	0.078	0.065	0.050	0.163	0.162	0.160	0.063	0.061	0.058

Table A4. The final results of the fuzzy Ratio System (RS) of MOORA

States	$\tilde{y}_i^*$			BNP_i	**Rank (RS)**
	y_{i1}^*	y_{i2}^*	y_{i3}^*		
BE	0.616	0.534	0.435	0.528	11
BG	–0.581	–0.378	–0.129	–0.363	27
CZ	0.408	0.403	0.429	0.413	13
DK	0.915	0.879	0.843	0.879	3
DE	0.697	0.650	0.565	0.638	8
EE	0.091	0.203	0.358	0.217	19
IE	0.642	0.607	0.569	0.606	9
EL	0.227	0.146	0.061	0.145	22
ES	0.341	0.326	0.317	0.328	14
FR	0.641	0.562	0.450	0.551	10
IT	0.315	0.283	0.234	0.277	16
CY	0.290	0.275	0.288	0.285	15
LV	0.036	0.217	0.372	0.208	20
LT	0.033	0.184	0.353	0.190	21
LU	0.998	0.995	0.984	0.992	2
HU	0.253	0.238	0.182	0.224	18
MT	0.044	0.034	0.015	0.031	23
NL	0.791	0.764	0.712	0.756	6
AT	0.829	0.802	0.781	0.804	5
PL	–0.141	–0.035	0.085	–0.030	24
PT	0.291	0.244	0.256	0.264	17
RO	–0.175	–0.055	0.069	–0.054	25
SI	0.467	0.459	0.456	0.460	12
SK	–0.128	–0.134	–0.061	–0.108	26
FI	0.865	0.846	0.804	0.838	4
SE	1.137	1.067	1.007	1.071	1
UK	0.762	0.701	0.631	0.698	7

Table A5. The fuzzy Reference Point (RP) of MOORA
A5a – Maximal Objective Reference Point:

j	1			2			3			4			5			6		
	r_{j1}	r_{j2}	r_{j3}	r_{j1}	r_{j2}	r_{j3}	r_{j1}	r_{j2}	r_{j3}	r_{j1}	r_{j2}	r_{j3}	r_{j1}	r_{j2}	r_{j3}	r_{j1}	r_{j2}	r_{j3}
$\tilde{r}$	0.465	0.468	0.477	0.349	0.344	0.335	0.234	0.230	0.225	0.327	0.307	0.280	0.228	0.226	0.224	0.449	0.428	0.405
j	7			8			9			10			11			12		
	r_{j1}	r_{j2}	r_{j3}	r_{j1}	r_{j2}	r_{j3}	r_{j1}	r_{j2}	r_{j3}	r_{j1}	r_{j2}	r_{j3}	r_{j1}	r_{j2}	r_{j3}	r_{j1}	r_{j2}	r_{j3}
$\tilde{r}$	0.251	0.244	0.249	0.086	0.091	0.100	0.108	0.112	0.116	0.039	0.042	0.043	0.076	0.079	0.081	0.057	0.051	0.045

A5b – Distances of responses from the reference point $d(\tilde{r}_j, \tilde{x}_{ij}^*)$:

j	1	2	3	4	5	6	7	8	9	10	11	12	max d	Rank (RP)
BE	0.251	0.084	0.047	0.171	0.027	0.205	0.054	0.132	0.059	0.164	0.108	0.053	0.251	6
BG	0.408	0.276	0.062	0.169	0.035	0.369	0.077	0.000	0.100	0.250	0.038	0.485	0.485	27
CZ	0.336	0.210	0.034	0.119	0.005	0.274	0.035	0.034	0.000	0.125	0.063	0.230	0.336	16
DK	0.242	0.133	0.000	0.044	0.048	0.142	0.067	0.194	0.022	0.000	0.105	0.000	0.242	4
DE	0.258	0.130	0.028	0.108	0.047	0.133	0.071	0.129	0.041	0.202	0.076	0.024	0.258	8
EE	0.369	0.234	0.037	0.069	0.031	0.325	0.006	0.048	0.123	0.148	0.012	0.271	0.369	19
IE	0.223	0.083	0.031	0.084	0.020	0.281	0.062	0.163	0.111	0.056	0.158	0.007	0.281	11
EL	0.308	0.148	0.053	0.122	0.029	0.361	0.072	0.095	0.136	0.214	0.158	0.037	0.361	17
ES	0.288	0.139	0.050	0.121	0.074	0.300	0.014	0.097	0.122	0.120	0.191	0.038	0.300	13
FR	0.268	0.100	0.040	0.152	0.026	0.184	0.077	0.136	0.066	0.143	0.106	0.031	0.268	10
IT	0.272	0.112	0.062	0.167	0.049	0.298	0.067	0.122	0.117	0.188	0.128	0.018	0.298	12
CY	0.303	0.174	0.026	0.072	0.030	0.388	0.098	0.097	0.080	0.002	0.271	0.055	0.388	24
LV	0.386	0.252	0.044	0.104	0.036	0.370	0.017	0.041	0.139	0.181	0.000	0.124	0.386	23
LT	0.380	0.240	0.046	0.099	0.023	0.347	0.066	0.029	0.120	0.169	0.006	0.182	0.380	21
LU	0.000	0.000	0.042	0.170	0.045	0.237	0.082	0.137	0.045	0.009	0.091	0.029	0.237	2
HU	0.362	0.215	0.061	0.185	0.023	0.325	0.058	0.031	0.038	0.130	0.071	0.158	0.362	18
MT	0.326	0.162	0.066	0.171	0.112	0.382	0.133	0.067	0.065	0.136	0.182	0.045	0.382	22
NL	0.231	0.118	0.007	0.106	0.046	0.226	0.078	0.125	0.023	0.018	0.114	0.035	0.231	1
AT	0.239	0.110	0.021	0.155	0.019	0.166	0.034	0.126	0.042	0.020	0.131	0.017	0.239	3
PL	0.376	0.226	0.065	0.176	0.007	0.358	0.072	0.035	0.109	0.252	0.087	0.154	0.376	20
PT	0.327	0.201	0.026	0.076	0.108	0.307	0.031	0.088	0.131	0.093	0.185	0.040	0.327	15
RO	0.409	0.273	0.050	0.115	0.040	0.375	0.055	0.014	0.131	0.139	0.036	0.304	0.409	25
SI	0.315	0.184	0.034	0.183	0.009	0.253	0.020	0.070	0.033	0.107	0.129	0.081	0.315	14
SK	0.361	0.209	0.053	0.179	0.000	0.364	0.022	0.028	0.038	0.433	0.050	0.258	0.433	26
FI	0.259	0.118	0.024	0.089	0.017	0.021	0.072	0.166	0.037	0.056	0.123	0.061	0.259	9
SE	0.243	0.118	0.011	0.000	0.018	0.000	0.095	0.162	0.014	0.019	0.099	0.027	0.243	5
UK	0.253	0.122	0.016	0.060	0.038	0.221	0.098	0.138	0.117	0.026	0.083	0.010	0.253	7

Annex B. The fuzzy Full Multiplicative Form and fuzzy MULTIMOORA

Table B1. The fuzzy Full Multiplicative Form (MF)

State	A_{i1}	A_{i2}	A_{i3}	B_{i1}	B_{i2}	B_{i3}	U'_{i1}	U'_{i2}	U'_{i3}	BNP_i	Rank (MF)
BE	6.33E+10	8.84E+10	1.26E+11	81229192	1.21E+08	1.65E+08	382.6751	733.2964	1.02E+19	3.41E+18	**2**
BG	3.98E+08	1.43E+09	5.41E+09	87525501	2.87E+08	8.61E+08	0.461969	4.977532	4.74E+17	1.58E+17	**23**
CZ	2.05E+10	3.47E+10	5.99E+10	32240603	80841425	1.59E+08	129.3744	429.2268	1.93E+18	6.44E+17	**17**
DK	1.41E+11	1.96E+11	2.71E+11	6216945	12792050	22284777	6346.014	15315.89	1.68E+18	5.61E+17	**18**
DE	8.7E+10	1.22E+11	1.88E+11	46374847	72006311	1.23E+08	707.2274	1693.321	8.71E+18	2.9E+18	**3**
EE	6.15E+09	1.8E+10	5.35E+10	44436590	1.55E+08	4E+08	15.38156	116.3191	2.38E+18	7.92E+17	**13**
IE	7.57E+10	1.18E+11	1.83E+11	37000885	55552085	76034200	995.5973	2122.429	6.76E+18	2.25E+18	**5**
EL	1.25E+10	1.77E+10	2.33E+10	1.25E+08	2.04E+08	3.01E+08	41.58832	86.78519	2.91E+18	9.69E+17	**10**
ES	2.55E+10	4.22E+10	6.98E+10	72131495	1.46E+08	2.55E+08	100.1222	289.2566	5.03E+18	1.68E+18	**6**
FR	6.46E+10	8.91E+10	1.18E+11	64165873	88772369	1.19E+08	540.4122	1003.993	7.59E+18	2.53E+18	**4**
IT	2.18E+10	3.22E+10	4.97E+10	81222166	1.28E+08	2.12E+08	102.5437	250.9532	4.04E+18	1.35E+18	**8**
CY	5.3E+09	1.09E+10	2.6E+10	24336703	49704816	81494291	65.04372	218.485	6.32E+17	2.11E+17	**20**
LV	2.27E+09	5.6E+09	1.44E+10	20061080	78102282	2.87E+08	7.879871	71.63826	2.9E+17	9.66E+16	**27**
LT	3.04E+09	8.08E+09	1.89E+10	17473448	92256740	2.99E+08	10.14279	87.61634	3.3E+17	1.1E+17	**25**
LU	1.31E+11	1.73E+11	2.32E+11	7108890	19116414	47732500	2735.72	9052.587	1.65E+18	5.5E+17	**19**
HU	4.71E+09	8.74E+09	1.37E+10	44043797	77979617	1.31E+08	35.94482	112.1021	6.04E+17	2.01E+17	**21**
MT	1.1E+09	2.69E+09	5.39E+09	60852740	1E+08	1.67E+08	6.612322	26.76281	3.28E+17	1.09E+17	**26**
NL	7.37E+10	1.12E+11	1.64E+11	14066815	22088363	36808511	2002.01	5065.027	2.3E+18	7.67E+17	**14**
AT	9.41E+10	1.39E+11	2.41E+11	15588698	21769174	33230329	2832.278	6391.682	3.75E+18	1.25E+18	**9**
PL	2.62E+09	4.66E+09	8.12E+09	66834784	2.26E+08	6.22E+08	4.21494	20.67204	5.43E+17	1.81E+17	**22**
PT	1.19E+10	1.94E+10	4.09E+10	62643874	1.21E+08	1.95E+08	61.22833	160.627	2.56E+18	8.54E+17	**11**
RO	5.61E+08	1.94E+09	7.88E+09	59996905	1.49E+08	4.01E+08	1.397506	13.02349	4.73E+17	1.58E+17	**24**
SI	2.29E+10	3.8E+10	6.38E+10	39929745	80673819	1.43E+08	159.785	471.6043	2.55E+18	8.49E+17	**12**
SK	3.36E+09	7.5E+09	2.04E+10	1.1E+08	2.89E+08	6.03E+08	5.56885	25.96284	2.24E+18	7.47E+17	**16**
FI	1.69E+11	2.38E+11	3.35E+11	34102820	69290315	1.39E+08	1219.223	3437.218	1.14E+19	3.81E+18	**1**
SE	2.78E+11	3.41E+11	4.15E+11	9840525	23355097	39945657	6960.107	14579.86	4.08E+18	1.36E+18	**7**
UK	8.52E+10	1.09E+11	1.37E+11	16670117	27764214	40198084	2120.383	3916.003	2.29E+18	7.62E+17	**15**

Annex C. Summary table for the three Methods of Fuzzy MULTIMOORA

Table C1. Final ranks of a fuzzy MULTIMOORA for EU member states (2000-2004-2008)

State	Ranks			Sum	Final rank by Sum MULTI-MOORA	Group by Sum	Rank Correc-tion by Domi-nance	Group Correc-tion by Domi-nance
	The Fuzzy Ratio System	The Fuzzy Reference Point	The Fuzzy Full Multiplicative Form					
Austria	5	3	9	17	3	1	4	–
Belgium	11	6	2	19	5	1	7	–
Bulgaria	27	27	23	77	27	3	–	–
Cyprus	15	24	20	59	20	3	19	–
Czech Republic	13	16	17	46	16	2	–	–
Denmark	3	4	18	25	10	2	6	1
Estonia	19	19	13	51	18	2	–	–
Finland	4	9	1	14	2	1	3	–
France	10	10	4	24	8	1	10	–
Germany	8	8	3	19	4	1	9	–
Greece	22	17	10	49	17	2	–	–
Hungary	18	18	21	57	19	3	20	–
Ireland	9	11	5	25	9	1	11	2
Italy	16	12	8	36	13	2	–	–
Latvia	20	23	27	70	24	3	–	–
Lithuania	21	21	25	67	22	3	–	–
Luxembourg	2	2	19	23	7	1	2	–
Malta	23	22	26	71	25	3	23	–
Netherlands	6	1	14	21	6	1	5	–
Poland	24	20	22	66	21	3	–	–
Portugal	17	15	11	43	15	2	-	–
Romania	25	25	24	74	26	3	25	–
Slovakia	26	26	16	68	23	3	26	–
Slovenia	12	14	12	38	14	2	–	–
Spain	14	13	6	33	12	2	–	–
Sweden	1	5	7	13	1	1	–	–
United Kingdom	7	7	15	29	11	2	8	1

Annex D. Theory of Dominance, Domination and Transitivity

1. Principles

1. Staying in the ordinal sphere with ranking by dominance.
2. The three methods have the same importance.
3. Overall dominance is ranked on the first place. Will seldom occur.
4. Three groups are considered: Core (in principle first 9), Semi-Periphery (next 9), Periphery (last 9). If countries are ex-aequo but a country is Semi-Periphery or Periphery in one of the methods then it is inferior to the other country.

2. Ranking

Overall dominance in the three methods is not present.

I. Core

1. General dominance in two of the three methods: Sweden (1-5-7)

– Dominates Luxemburg (2-2-19) in:	1) Ratio System; Dominated in Reference Point. 2) Multiplicative Form.
– Dominates Austria (5-3-9) in:	1) Ratio System; Dominated in Reference Point. 2) Multiplicative Form.
– Dominates Finland (4-9-1) in:	1) Ratio System; Dominated in Multiplicative Form. 2) Reference Point.

– Dominates all the others in 2 methods

2. Dominance in two of the three methods: Luxemburg (2-2-19)

3. Dominance in two of the three methods: Finland (4-9-1) dominated by Luxemburg. Dominates Austria in 2 methods.

4. Austria (5-3-9)	2 x dominated by Finland.
5. Netherlands (6-1-14)	2 x dominated by Austria.
6. Denmark (3-4-18)	2 x dominated by the Netherlands.
7. Belgium (11-6-2)	2 x dominated by Denmark.
8. UK (7-7-15)	2 x dominated by Belgium.
9. Germany (8-8-3)	2 x dominated by UK.

II. Semi-Periphery

10. France (10-10-4)	overall dominated by Germany.
11. Ireland (9-11-5)	2 x dominated by France.
12. Spain (14-13-6)	overall dominated by Ireland.
13. Italy (16-12-8)	2 x dominated by Spain.
14. Slovenia (12-14-12)	2 x dominated by Italy.
15. Portugal (17-15-11)	2 x dominated by Slovenia.
16. Czech (13-16-17)	2 x dominated by Portugal.
17. Greece (22-17-10)	2 x dominated by Czech Republic.
18. Estonia (19-19-13)	2 x dominated by Greece.

III. Periphery

19. Cyprus (15-24-20)	2 x dominated by Estonia.
20. Hungary (18-18-21)	2 x dominated by Cyprus.
21. Poland (24-20-22)	2 x dominated by Hungary.
22. Lithuania (21-21-25)	2 x dominated by Poland.
23. Malta (23-22-26)	overall dominated by Lithuania.
24. Latvia (20-23-27)	2 x dominated by Malta.
25. Romania (25-25-24)	2 x dominated by Latvia.
26. Slovakia (26-26-16)	2 x dominated by Romania.
27. Bulgaria (27-27-23)	overall dominated by Slovakia.

NERAIŠKIŲJŲ SKAIČIŲ TEORIJA PAPILDYTAS *MULTIMOORA* METODAS EUROPOS SĄJUNGOS VALSTYBIŲ NARIŲ IŠSIVYSTYMO VERTINIMUI

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Santrauka. Neraiškioji logika padeda įvertinti ir spręsti neapibrėžtas problemas įvairiose srityse. Neraiškieji skaičiai gali išreikšti tiek kiekybinius, tiek kokybinius kintamuosius. Kiekybiniai neraiškieji kintamieji gali apimti tradicinius realiuosius skaičius, susintetintus istorinius duomenis (laiko eilutes) ar prognozuojamas tendencijas. Kokybiniai neraiškieji kintamieji gali būti naudojami dirbant su rangų skalėmis (lingvistiniai kintamieji). Taigi daugiakriterinio vertinimo metodų praplėtimas neraiškiųjų skaičių aibių teorija yra svarbus klausimas. MULTIMOORA metodas buvo papildytas neraiškiųjų skaičių teorija. Viršūnės metodas pritaikytas skaičiuojant atstumus tarp neapibrėžtųjų skaičių. Ploto centro metodas pritaikytas konvertuojant neraiškiuosius skaičius į realiuosius. MULTIMOORA metodą sudaro trys dalys: santykių sistema, atskaitos taškas ir pilnoji sandaugos forma. Kiekviena dalis buvo modifikuota papildant ją trečiojo laipsnio neraiškiaisiais skaičiais. Neraiškioji santykių sistema apima vidinį normalizavimą, kriterijų apibendrinimą ir konvertavimą į apibrėžtuosius skaičius. Neraiškioji atskaitos taško sistema remiasi atskaitos taško (vektoriaus) nustatymu ir kiekvienos alternatyvos atstumo iki jo matavimu taikant viršūnės metodą. Neraiškioji pilnoji sandaugos forma sujungia grynosios multiplikatyvinės naudingumo funkcijos maksimizavimą ir konvertavimą į realiuosius skaičius. Neraiškusis MULTIMOORA metodas apibendrina šiuos tris požiūrius. Straipsnyje išspręsta rangų apibendrinimo problema, iškylanti apibendrinant keliais daugiakriterinio optimizavimo metodais gautus rangus. Šiam tikslui pasiūlyta ir pritaikyta dominavimo teorija, apibūdinanti įvairias alternatyvų palyginimo procedūras remiantis skirtingais tos pačios alternatyvos rangais. ES valstybių narių pažanga įgyvendinant Lisabonos strategijos tikslus 2000–2008 m. įvertinta taikant neraiškųjį MULTIMOORA metodą ir dominavimo teoriją. Analizės rezultatai rodo, kad pirmauja Švedija, Liuksemburgas, Suomija, Austrija, Nyderlandai, Danija, Belgija, Jungtinė Karalystė ir Vokietija. Antrajai grupei priklauso Prancūzija, Airija, Ispanija, Italija, Slovėnija, Portugalija, Čekija, Graikija ir Estija. Labiausiai atsilieka Vengrija, Kipras, Lenkija, Lietuva, Slovakija, Latvija, Malta, Rumunija ir Bulgarija.

Reikšminiai žodžiai: daugiakriterinis optimizavimas, MOORA, MULTIMOORA, struktūriniai rodikliai, Lisabonos strategija, strateginis valdymas, Europos Sąjunga, darnus vystymas, neraiškieji skaičiai, trečiojo laipsnio skaičiai, dominavimo teorija, tranzityvumas.

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