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Review article

PAIRWISE COMPARISON MATRIX IN MULTIPLE CRITERIA DECISION MAKING

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Abstract. The measurement scales, consistency index, inconsistency issues, missing judgment estimation and priority derivation methods have been extensively studied in the pairwise comparison matrix (PCM). Various approaches have been proposed to handle these problems, and made great contributions to the decision making. This paper reviews the literature of the main developments of the PCM. There are plenty of literature related to these issues, thus we mainly focus on the literature published in 37 peer reviewed international journals from 2010 to 2015 (searched via ISI Web of science). We attempt to analyze and classify these literatures so as to find the current hot research topics and research techniques in the PCM, and point out the future directions on the PCM. It is hoped that this paper will provide a comprehensive literature review on PCM, and act as informative summary of the main developments of the PCM for the researchers for their future research.

Keywords: Analytical Hierarchy Process (AHP), Analytical Network Process (ANP), consistency, pairwise comparison matrix (PCM), missing judgment estimation, priority derivation, multi-criteria decision-making (MCDM).

JEL Classification: C44, D7, D81.

Introduction

The pairwise comparison technique has been widely used to tackle the subjective and objective judgments about qualitative and/or quantitative criteria in multi-criteria decision making (MCDM), especially in the Analytical Hierarchy Process (AHP) and Analytical Network Process (ANP), and usually denoted as pairwise comparison matrices (hereinafter, PCMs). The preference relations in the PCMs are filled in by the decision maker judgments, and presented using different measurement scales such as ratio scale (Saaty 1977), geometric

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scale (Lootsma 1989) and logarithmic scale (Ishizaka *et al.* 2010) etc. The judgments may be inconsistent and/or incomplete because of the limits of decision makers' expertise and capabilities or the complexity of the decision problems, and various approaches and models are proposed to handle these problems (Benítez *et al.* 2011; Ergu *et al.* 2011; Kou *et al.* 2014a). To evaluate the level of inconsistency in a PCM, different consistency indices have been proposed and compared (Brunelli *et al.* 2013a). The importance of criteria and the ranking of alternatives are often judged through the priority weights derived from a PCM, thus many prioritization approaches have been proposed to derive the priority weights from a PCM (Cavallo, D'Apuzzo 2011; Kou *et al.* 2014b).

There is an article reviewing the main developments in the AHP regarding the problem modelling, pairwise comparisons, judgment scales, derivation methods, consistency indices, incomplete matrix, synthesis of the weights, sensitivity analysis and group decisions from 1996 to 2010 (Ishizaka, Labib 2011). This paper extends some of the main developments through a literature review, and mainly focuses on the hot research topics in the PCM, including measurement scales, cardinal or ordinal inconsistency processing models, missing data estimation, consistency index and priority derivation methods. The reviewed literature were published in 37 peer reviewed international journals and searched via the ISI Web of science from 2010 to 2015. Based on the 93 journal articles collected, we attempt to analyze which topics on PCM were prevalently studied in recent years and which techniques were used to explore these issues, while which directions on the PCM remain to be studied in future. In addition, it is hoped that this paper will provide a comprehensive literature review on PCM, and act as informative summary of the main developments of the PCM for the researchers for their future researches.

The paper is organized as follows: Section 1 reviews the literature on the measurement scales, consistency indices, inconsistency processing models, missing judgments processing models and priority derivation methods in the multiplicative and additive PCM as well as interval PCM. The literature on Statistical (stochastic) approaches are reviewed in this section. Section 2 analyses the most prevalently research topics in the PCM, and some observations and discussions are presented in this section. The last Section concludes the paper.

1. Review of the main research topics in the PCM

When it comes to the pairwise comparison matrix (PCM), the existing researches usually focus on the measurement scales, consistency index, inconsistency issues and priority derivation methods either in multiplicative, additive or interval (fuzzy) PCM, therefore, the reviews of the main research topics in the PCM are grouped into two classes, i.e. multiplicative and additive PCM approach, interval PCM approach. Most of these researches were conducted based on optimization methods. Few of the articles focused on these issues from the statistical perspective, therefore, a review on the literature from the statistical perspective is presented separately in the following.

1.1. Multiplicative and additive PCM approaches

1.1.1. Measurement scales

Five out of ninety-three articles (5.38%) studied the measurement scales in the PCM.

Fülöp *et al.* (2010) focused on small scale construction for the PCM, and proposed to use the 3-point scale (1 to 3 scale). They proved that the smaller scale has better mathematical foundations than larger ones. Kim *et al.* (2010) analyzed the consistency concept, and the 9-point scale was mapped to verbal scale. Based on the verbal scale, they proposed new criteria to solve the contradictory transitivity. The Monte-Carlo simulation based on bootstrap approach was conducted to derive the criteria. Choo, Wedley (2010) analyzed the issue of value changes between different unit measure when multiplying a ratio scale by a positive constant, and evaluated column averaging approaches for aggregating estimates with unknown units into overall values.

Dong *et al.* (2013) presented a novel framework for AHP users to generate numerical scales individually, which is based on the 2-tuple linguistic modeling of AHP scale problems.

Xia *et al.* (2015) presented the relative measure to calculate the relative values based on Abelian linearly ordered group (Alo-group).

1.1.2. Consistency indices

In a PCM, the relationship between judgments might be multiplicative, additive or fuzzy. For the different relationship, the consistency indices are different, and many consistency indices have been proposed to measure the consistency for a PCM. Among the reviewed 93 articles, 10 papers (10.75%) focused on the inconsistency indices. Cavallo and D'Apuzzo (2010) presented an alo-group $G = (G, \odot, \leq)$ to unify the multiplicative, additive and fuzzy PCMs and a consistency index $IG(A)$ is developed to measure the consistency for the generated PCM.

Brunelli *et al.* (2013a) analyzed ten inconsistency indices of PCMs by numerical examples to investigate the degrees of their agreement. They are: *CI*, Index of determinants and *c*3 (CI^*, $-c_3$), Squared differences index (*LS)*, the Geometric Consistency Index (*GCI*), Harmonic Consistency Index (*HCI*), Cavallo and D'Apuzzo (I_{CD}), the relative error index (*RE*), Golden-Wang index (*GW*), Koczkodaj index (*K*), Ramík-Korviny index (NI_n^{σ}). Brunelli *et al.* (2013b) analyzed four consistency indices in the AHP in order to avoid redundancy in the selection of consistency preferences. The compared indices are: the Geometric Consistency Index (*GCI*), the index of Lamata and Peláez (CI^*), the index c_3, the index ρ.

Ergu *et al.* (2014a) presented a maximum eigenvalue threshold as the consistency index for the ANP, and a block diagonal matrix was introduced to reduce the times of consistency test, then the inconsistent elements were identified and adjusted by an induced bias block diagonal comparison matrix.

Brunelli, Fedrizzi (2015a) analyzed the relationship between the consensus and preference aggregation of the PCM in group decisions. The following inconsistency indices, *CI*, $CI^{*,}$ *GCI,* I_{CD} and *K*, were analyzed and some properties of the inconsistency indices were defined. Brunelli, Fedrizzi (2015b) presented five axioms to characterize inconsistency indices, and some of the existing indices were tested by the proposed axioms. They found some of the inconsistency indices satisfy the proposed axioms while some are not. Meng, Chen (2015) proposed the multiplicative geometric consistent index (MGCI) to measure the consistency of multiplicative preference relations. Koczkodaj *et al.* (2016) presented an abelian linearly ordered group (alo-group) to analyze the iconsistency indicator map of PCM on a group, and proposed a new consistency index that called T-inconsistency indicator. Siraj *et al.* (2015) investigated various consistency measures by Monte-Carlo simulations, and proposed two measures (i.e. congruence and dissonance) for cardinal inconsistency and ordinal inconsistency. Xia, Chen (2015b) defined a consistency index $I_{\Omega}(A)$ for the PCM generated by Abelian linearly ordered group (alo-group), and consistency improving method has been proposed to deal with the inconsistency. Meanwhile, a consensus index $GI_{\Omega}(A)$ was defined to aggregate the individual PCMs. Grzybowski (2016) investigated the relationships among the values of the consistency indices, the "consistency" of the decision maker's judgments and prioritization results. The Monte Carlo simulations were conducted to analyze the performance of the most common inconsistency indices, and a new inconsistency index, called the average value of all "triad inconsistencies" (ATI) was proposed based on the *KI* index.

1.1.3. Inconsistency processing models

1.1.3.1. Ordinal inconsistency

The intransitivity or contradictory phenomenon between paired comparisons is regarded as ordinal inconsistency. four out of reviewed ninety-three papers (4.3%) studied the ordinal inconsistency in a PCM.

Kéri (2010) employed the graph theoretic approach to deal with the intransitive and contradictory judgment matrices.

Siraj *et al.* (2012a) developed a heuristic algorithm to improve the ordinal consistency through identifying and eliminating the intransitive judgments in the PCMs. The near-optimal solution could be generated by the proposed model. In addition, Monte-Carlo simulation was conducted to collect the statistical evidence on the occurrence of three-way cycle in a PCM with acceptable consistency ratio.

Kou *et al.* (2014a) proposed a Hadamard product induced bias matrix (HPIBM) model to handle the cardinal and ordinal inconsistencies simultaneously. The proposed model is only based on the original matrix and is independent of the methods chosen to the prioritization methods.

Cavallo, D'Apuzzo (2015) analyzed the transitivity issue in the PCM over alo-group, and a tool was proposed to check the transitivity in the alo-group based PCM.

1.1.3.2. Cardinal inconsistency

Compared with the ordinal inconsistency, the cardinal inconsistency happened more often in the real world decision making, and much attention has been paid to this issue. Among the reviewed 93 papers, 17 papers (18.28%) focused on the cardinal inconsistency issue, which are reviewed below.

Temesi (2010) discussed the relationship between the consistency of a PCM and the consistency of the decision maker. The error-free property is proposed to describe the latter consistency.

Benítez *et al.* (2011) presented a linearization technique to handle the inconsistency issue in a PCM. Broadly speaking, the orthogonal projection is used to obtain the closest consistent matrix to an original inconsistent matrix, and the consistency has therefore been improved in a closed form. Ergu *et al.* (2011) proposed an induced bias matrix model to identify the most inconsistent elements in the PCM. The proposed model is capable of preserving most of the original information provided by the decision makers. To avoid the consistency issue and reduce the times of pairwise comparisons in a PCM, Hsu, Wang (2011) established a multi-criteria decision making with incomplete linguistic preference relations model (InLinPreRa) by using horizontal, vertical and oblique pairwise comparisons algorithm. In the proposed model, only *n-1* pairwise comparisons need to be provided instead of *n(n-1)/2* times. Hou (2011) applied semirings algebra to discuss the properties of multiplicative reciprocal judgment matrices and additive reciprocal judgment matrices. In the proposed approach, optimization models were developed to find the nearest consistent judgment matrx.

Benítez *et al.* (2012) proposed an optimization method to improve the consistency of PCMs. The proposed model is based on the minimization of the distance between two matrices, and the number of decision variables is reduced to improve the computational efficiency.

Bozóki *et al.* (2013) conducted an empirical research on the empirical PCMs. The CR index proposed by Saaty and the CM proposed by Koczkodaj index were used to test the consistencies for the PCMs. They found that two factors impacted on the inconsistency, i.e. the type of the problem and the size of the matrix. In addition, They investigated the incomplete matrices so as to reveal the decision makers' behavior during the completing process.

For the dimensionality issue, Jalao *et al.* (2014a) proposed a PCM decomposition methodology to reduce the number of pairwise comparisons. The binary integer programming was used to decompose the PCM into smaller subsets, and the local priorities and the pivot element were obtained by minimizing the inner dependencies to estimate the global priorities. Ergu *et al.* (2014b) conducted simulation experiments for improving the consistency ratio of PCMs. The simulation was based on an induced bias matrix model. Zhang, H. *et al.* (2014) first defined the modified consistent PCM and an adjustable consistent PCM using the original inconsistent matrix, then developed an algorithm with *segment tree* to derive a consistent PCM with crisp or fuzzy elements. Based on three inconsistency indices (CR, CM and CI), Bozóki *et al.* (2014) employed a nonlinear mixed-integer optimization approach to find the minimal number of matrix elements in order to obtain appropriate modification and make the matrix acceptable. In addition, the proposed model can improve the consistency given the maximal number of modifiable matrix elements. Pereira, Costa (2014) presented a nonlinear programming model to improve the inconsistency by adjusting the original judgments in a minimum way. Girsang *et al.* (2014) proposed an ant algorithm based approach to find the minimal distance between the original PCM and the modified PCM in the AHP.

Kułakowski *et al.* (2015) proposed a concurrent inconsistency reduction algorithm to obtain a generalized PCM, aiming to deal with the large order of matrices in large decision support systems. Koczkodaj *et al.* (2015) conducted theoretical proof and empirical evidence of the reduction algorithm convergence for the distance-based inconsistency in the PCM. The Monte Carlo simulation was conducted to demonstrate the convergence speed of inconsistency reduction in pairwise comparisons. Xia, Chen (2015a) introduced the bilateral agreement to conduct group evaluation of alternatives, and employed the quasi-arithmetic mean to ensure the consistency property of the PCMs in multi-criteria group decision making.

Zhang (2016) studied the properties of the consistency and consensus of multiplicative consistent reciprocal preference relations, and a consensus optimization model for group decision making was proposed to obtain consensus with the highest overall consensus level.

1.1.4. Priority derivation methods

How to derive the priority vectors from a PCM is one of the most important issues, and many prioritization methods have therefore been proposed. Ten out of reviewed ninety-three papers (10.75%) studied the priority derivation methods.

Fedrizzi, Brunelli (2010) presented two straightforward methods for deriving the priority vector in the additively consistent PCM and multiplicatively consistent PCM respectively, and analyzed the relationships between the weight vectors and the reciprocal relations. Yuen (2010) proposed the analytic hierarchy prioritization process (AHPP) to provide the guidelines for selecting the most appropriate prioritization operator when the PCM is inconsistent. Nine prioritization operators and seven measurement criteria were used to validate the effectiveness of the proposed model.

Huo *et al.* (2011) developed a new parametric prioritization method (PPM) by three parameters to derive the priority vectors from a PCM and proved that a consistent complementary matrix can be transformed into a consistent reciprocal matrix, vice versa. Cavallo, D'Apuzzo (2011) applied the Abelian linearly ordered group (alo-group) to derive weights from a PCM. The proposed model satisfied the independence of scale-inversion condition. Dijkstra (2011) studied the properties of weight extraction methods for the PCMs by minimizing the suitable measures of inconsistency, "average error gravity", and recommended the geometric mean when considering the weight extraction.

Lin *et al.* (2013a) developed a logarithmic transformation based algorithm to obtain a nearer consistent matrix so as to derive the priority vector.

Kou, Lin (2014) proposed a cosine maximization method (CM) that is based on similarity measure to derive the priority vector for a PCM. The proposed model maximizes the sum of the cosine of the angle between the priority vector and each column vector of a PCM, and then reliable priority vector can be derived.

Tomashevskii (2015) analyzed the reliability of the eigenvector method (EM) based on "right–left asymmetry", "rank reversal" and reversal of "order of intensity of preference". This study shows that the numerical value of the errors completely relies on the inaccuracy of a measuring scale and inconsistent judgments. Jablonsky (2015) compared the three popular prioritization methods of PCM such as eigenvector method (EM), LLSM, LSM with other three goal programming methodologies, minimization of the sum of absolute and relative deviations (ASUM and RSUM) and minimization of the maximum deviation (absolute AMAX and relative RMAX). Kułakowski (2015a) analyzed the relationship between inconsistency of input and discrepancy of output, and two properties of the prioritization procedures were proposed based on the inconsistency and discrepancy indices.

1.1.5. Missing judgments processing models

In the decision making problem, incomplete judgments could occur because of various factors such as the incomplete information and limited expertise etc, thus the missing judgment estimation, the consistency issue of incomplete pairwise matrix have been paid more attention to in the PCM. Eight out of reviewed ninety-three papers (8.6%) concentrated on the missing estimation issue.

Gomez-Ruiz *et al.* (2010) developed a model based on the Multi-Layer Perception (MLP) neural network to estimate the missing judgments in an incomplete PCM, and improve its consistency simultaneously.

Dopazo, Ruiz-Tagle (2011) defined a similarity function and a parametric compromise function to develop a logarithmic goal programming formulation computational method for incomplete PCMs in the group decision-making problem. Bozóki *et al.* (2011) extended the distance-based inconsistency indicator to the incomplete case in a PCM. They transformed the optimization problem into an equivalent linear programming problem so as to obtain an optimal solution.

Siraj *et al.* (2012b) proposed generation of all possible preferences from a set of PCMs. Based on a graph-theoretic approach, the *pivotal combination* concept was introduced to generate a forest with all spanning trees. In the proposed model, the following three factors such as the *mean* of all preferences, the *variance*, the *enumerating all spanning trees* (EAST) were used to deal with the final priority vector, the inconsistency measurement, the preferences estimation in an incomplete PCMs, respectively.

Fedrizzi, Giove (2013) proposed an optimal sequencing approach for incomplete PCMs in the case of large-dimensional problems. The fair involvement and the consistency of judgment were regarded as two criteria to define the choice rule and to obtain a rational questioning process.

Benítez *et al.* (2014) developed an approach to complete the incomplete judgments by minimizing the Frobenius norm based matrix distance.

Chen *et al.* (2015) proved that the connecting path method (CPM) can guarantee minimal geometric consistency index, and proposed a PCM based method to estimate the missing judgments whilst improve the consistency for an incomplete PCM. Ergu *et al.* (2016) proposed a revised geometric mean induced bias matrix to estimate the missing values for the incomplete decision matrix in the case of emergency management. The consistency ratio can be efficiently improved by the proposed model.

1.2. Interval PCM approaches

In addition to the multiplicative and additive judgment matrices, the interval judgment matrices are used to establish the decision maker's preference relations based on interval values.

1.2.1. Measurement scales

There are only three papers (3.23%) studied the measure scales. They are:

Dong *et al.* (2011) analyzed the individual numerical scale in the AHP, and a 2-tuple fuzzy linguistic model was proposed to evaluate the effect of the numerical scales.

Abdullah, Najib (2014) proposed a new preference scale by considering he membership function, the non-membership function and the degree of hesitation of interval-valued intuitionistic fuzzy numbers (IVIFN) simultaneously, and a modified interval-valued intuitionistic fuzzy weighted averaging was presented to define the weight entropy of the aggregated matrix of IVIFN.

Dong, Herrera-Viedma (2015) proposed a consistency-driven methodology to set the interval numerical scale without the need of the semantics used in interval type-2 fuzzy sets.

1.2.2. Consistency indices and Inconsistency processing models

Seven papers (7.53%) paid attention to the consistency indices and inconsistency issues in interval PCM.

Conde, de la Paz Rivera Pérez (2010) established an "interval judgment matrix" by determining a set of bounds on the preference ratios for the PCM, and a linear optimization problem was introduced to define a consistency index for the interval matrix, then the relative weights were derived by solving the linear optimization problem.

Pedrycz, Song (2011) applied the information granularity to investigate the consistency and the consensus of the individual PCMs in AHP based group decision making. The granular entries in the granular PCM was presented by intervals, and the inconsistency indices were minimized to increase the level of consensus within the group.

Dong *et al.* (2014) studied the consistency issues in interval PCMs, and a new consistency index of interval PCM was proposed based on logarithmic Manhattan distance, then linear programming models were presented to calculate the consistency indices for an interval PCM. A LP-based consistency improving model was also proposed for improving the consistency of interval PCMs.

Ramík (2015) employed the abelian linearly ordered group (alo-group) to handle the PCM with fuzzy entries, and two consistency indices were proposed to deal with the inconsistency of triangular fuzzy numbers (PCFN) matrices. Li *et al.* (2016) focused on the consistency ratio for interval multiplicative comparison matrices (IMCMs), and a geometric mean based index was proposed to test the indeterminacy ratio of an IMCM.

Wang (2015a) proposed a new triangular fuzzy arithmetic based transitivity equation to define consistent Triangular Fuzzy Preference Relation (TFPR), and an acceptable consistency was proposed for TFPRs. The normalized triangular fuzzy multiplicative weights were transformed into consistent TFPRs by geometric mean and uncertainty ration based transformation formulae. The weight vectors of TFPR were derived by a logarithmic least square model.

1.2.3. Priority derivation methods

There are 11 papers (11.83%) studying the priority derivation methods.

Torabi, Rafiei (2012) developed a single-decision-making optimization model along with two group-decision-making optimization model to derive the weights from fuzzy PCMs.

Xu, Cai (2012) studied group decision making problems with interval multiplicative preference relations, and proposed two linear programming models to derive the weight from intervals multiplicative preference relations. Then, the continuous ordered weighted averaging operator or the continuous ordered weighted geometric operator was used to aggregate all the values in each weight interval.

Mirhedayatian *et al.* (2013) proposed a new approach for ranking the alternatives in fuzzy AHP by fuzzy data envelopment analysis. Lin, J. *et al.* (2013) proposed a new formula for ranking multiplicative interval weights in the AHP, and an approximation and adjustment (AAM) method was presented to obtain multiplicative triangular fuzzy weights. The geometric mean centroid of multiplicative triangular fuzzy weight was proposed to compare two multiplicative triangular fuzzy weights. Izadikhah (2013) employed the ranking function to transform the triangular fuzzy data into crisp one, then proposed the goal programming method to derive the fuzzy weights of criteria from fuzzy PCM.

Mohtashami (2014) proposed a Modified Fuzzy Logarithmic Least Square Model (MFLLSM) to derive the crisp priority vector from consistent and inconsistent fuzzy PCMs. The triangular shaped fuzzy number and trapezoidal shaped fuzzy numbers were used to present the fuzzy judgments for the first time. Zhang, F. *et al.* (2014) proposed an algorithm to derive the final priority interval weights for both consistent and inconsistent interval PCM.

Ramík (2015) studied the fuzzy PCM by Abelian linearly ordered group (Alo-group), and some concepts on the reciprocity and consistency as well as priority vector for fuzzy PCMs were generalized and analyzed. Chen, Xu (2015) presented a new fuzzy programming method (NFPM) to derive the priority vector from an interval PCM. Dutta, Guha (2015) proposed a novel approach to derive weights from the PCM with intuitionistic fuzzy numbers (IFNs), which generates crisp priority from PCM with IF. Meng *et al.* (2015) proposed two new methods to derive the interval priority vector from the interval preference relations based on the eigenvalue method and the row geometric mean method.

1.2.4. Missing judgments processing models

Five papers (5.38%) proposed new approaches for estimating the missing judgments in the interval PCM.

Liu *et al.* (2012) proposed a goal programming model to complement the acceptable missing values in incomplete interval multiplicative preference relations (IMPR), which was based on the consistency property of IMPR, then a new algorithm was developed to obtain the priority vector from incomplete IMPR. Again, an interval weighted geometric averaging (IWGA) operator was proposed to aggregate the individual preference relations.

Wang, Chen (2014) focused on the consistency prioritization and completion of interval fuzzy preference relations, then a geometric mean based uncertainty ratio, a logarithmic least squares based method and a logarithm least squares completion approach were proposed to deal with the uncertainty, interval weights, inconsistency modification and missing values estimation for interval fuzzy preference relation (IFPR) respectively. Ramík (2014) studied the relations between transitivity and consistency of fuzzy PCMs and multiplicative preferences PCMs. A new approach was proposed to estimate the missing values in the fuzzy PCMs. Xu *et al.* (2014) focused on the incomplete interval fuzzy preference relation, and a new approach was proposed to handle the AHP for group decision making with incomplete IFPR.

Zhang *et al.* (2015) defined the concept of additive consistent hesitant fuzzy preference relations, and the following three concepts were introduced, i.e., incomplete hesitant fuzzy preference relation, acceptable incomplete hesitant fuzzy preference relation, and additive consistent incomplete hesitant fuzzy preference relation, then two estimation procedures were proposed to estimate the missing values in the incomplete hesitant fuzzy preference relation.

1.3. Statistical (stochastic) approaches

In recent years, some statistical or stochastic approaches were developed to measure the consistency level of the PCM and derive the priority weights from the PCM. However, only thirteen papers (13.98%) out of the ninety-three reviewed papers investigated the consistency indices, inconsistency issues and priority derivation methods from the statistical analysis perspective, very few articles have addressed the missing judgment estimation from the statistical analysis perspective.

1.3.1. Consistency indices and Inconsistency processing models

Five papers (5.38%) focused on the consistency indices and inconsistency processing models.

Tsyganok (2010) analyzed the effectiveness of some methods of expert estimate aggregation in the PCM based on the simulation of possible expert errors. The genetic algorithm was suggested to search the maximum possible deviation in the PCM.

Liu *et al.* (2011) proposed a method for solving the stochastic multiple criteria decision making (SMCDM) problem. The dominance degree matrix was constructed by comparisons of probability distributions, and PROMETHEE II was used to built an overall dominance degree matrix as so to obtain the final ranking order of alternatives.

Entani, Sugihara (2012) defined the uncertainty indices for intervals from the perspectives of entropy in probability, sum or maximum of widths, or ignorance, then obtained the intervals of attributes by minimizing the uncertainty indices.

Lin *et al.* (2013b) developed an improved statistical approach for consistency test of the PCM by combining the test hypotheses and maximum likelihood estimation. Based on the significance level.

Lin *et al.* (2014) proposed a new statistical approach to deal with consistency issue in a PCM based on the hypothesis test and the random consistency index. The proposed approach is not only capable of identifying the deviation of consistency index (CI), but also reflecting the significance level of testing the consistency.

1.3.2. Priority derivation methods

There are eight papers (8.6%) using statistical analysis approaches for deriving the weights from the PCM.

Bernasconi *et al.* (2010) transformed a rigorous statistical analysis equation about the ratio scale of the PCM in AHP into a regression model, and then a method of the statistical analysis was conducted to estimate the priority weights in the AHP that takes into account the distortions caused by the subjective weighting function. Zhang *et al.* (2010) proposed a novel method based on the stochastic dominance degree (SDD) to solve a discrete stochastic multiple criteria decision-making (MCDM) problem, and an approach based on PROMETHEE-II was proposed to derive the priority weights for ranking alternatives.

Jalao *et al.* (2014b) used the method-of-moments methodology to fit the varying stochastic preferences of the DM into beta stochastic pairwise comparisons. Zhu and Xu (2014) proposed numerical preference relations (NPRs) to be the general form of the four existing preferences relations, i.e. multiplicative preference relations (MPRs), fuzzy preference relations (FPRs), interval MPRs (IV-MPRs) and interval FPRs (IV-FPRs). Then a stochastic preference analysis (SPA) method was developed to aid the decision makers (DMs) in decision making.

Lin, Kou (2015) proposed a Bayesian revision method for improving the individual PCMs by making full use of the prior distribution for parameters and sample information. Wang (2015b) defined a geometric mean based uncertainty index to measure the uncertainty level of the established interval matrix. In addition, a parameterization approximate relation was presented to show the relation between the normalized interval probabilities and the established interval matrix. Then a two-stage procedure was proposed to obtain the interval probabilities from A multiplicative reciprocal comparison matrix. Kułakowski (2015b) presented a new iterative heuristic rating estimation algorithm that tries to deal with the situation when exact estimations for some concepts (stimulus) CK are a priori known and fixed. Yaraghi *et al.* (2015) used a simulation approach to compare the results of AHP with Monte Carlo analytic hierarchy process (MCAHP) under different levels of uncertainty. The results showed that the performance of AHP is not statistically different from the performance of MCAHP if the variation in different PCM is lower than 0.24, otherwise the MCAHP provides more precise rankings.

2. Observations and discussion

In the previous reviews, 93 journal articles published in 37 peer reviewed international journals from 2010 to 2015, were collected through ISI web of science. These articles mainly focused on the main developments in the PCM, including the measurement scales, consistency indices, inconsistency processing models, priority derivation methods, missing judgments processing models. Table 1 shows the distribution of the reviewed articles by journals. Obviously, the journal *European Journal of Operational Research* contains the most relevant articles, comprising 15 out of the 93 articles reviewed (16.13%), followed by *Information Sciences* (9.68%) and *Annals of Operations Research* (6.45%), while 21 journals contains only 1 related literature respectively.

Some observations based on the reviews are made and discussed in the following.

As classified in the previous sections, the pairwise comparison matrices can be generally grouped into three types: multiplicative, additive and interval (fuzzy) PCM.

Table 1. Distribution of the selected articles by journals

	Name of the journal	Amount (%)	Percentage
1	European Journal of Operational Research	15	16.13
2	Information Sciences	9	9.68
3	Annals of Operations Research	6	6.45
4	Applied Soft Computing	5	5.38
5	Expert Systems with Applications	5	5.38
6	Central European Journal of Operations Research	4	4.3
7	Applied Mathematics and Computation	4	4.3
8	Computers & Industrial Engineering	4	4.3
9	Soft Computing	3	3.23
10	Applied Mathematical Modelling	3	3.23
11	Fuzzy Sets and Systems	3	3.23
12	Mathematical and Computer Modelling	3	3.23
13	Computers & Operations Research	2	2.15
14	Fundamenta Informaticae	2	2.15
15	IEEE Transactions On Fuzzy Systems	2	2.15
16	International Journal of Intelligent Systems	2	2.15
17	Artificial Intelligence and Soft Computing	1	1.08
18	Abstract and Applied Analysis	1	1.08
19	Group Decision and Negotiation	1	1.08
20	International Journal of General Systems	1	1.08
21	International Journal of Computers Communications & Cont	1	1.08
22	IEEE Transactions on Cybernetics	1	1.08
23	IEEE Transactions on Engineering Management	1	1.08
24	Information Fusion	1	1.08
25	International Journal of Approximate Reasoning	1	1.08
26	International Journal of Computational Intelligence Systems	1	1.08
27	Journal of Intelligent & Fuzzy Systems	1	1.08
28	Journal of Physics: Conference Series	1	1.08
29	Journal of the Operational Research	1	1.08
30	Journal of the Korean Institute of Industrial Engineers	1	1.08
31	Journal of the Operational Research Society	1	1.08
32	Mathematical Problems in Engineering	1	1.08
33	Management Science	1	1.08
34	Neural Computing and Applications	1	1.08
35	Technological and Economic Development of Economy	1	1.08
36	The International Journal of Advanced Manufacturing Technology	1	1.08
37	Transactions on Computational Collective Intelligence	1	1.08
	Total	**93**	

It can be seen from Figure 1 that the researches on the first two types of PCM (54 papers, 58%) were more popular than the interval (fuzzy) PCM (26 papers, 27.96%). Moreover, 13 papers utilized the statistical analysis methods to study the above issues, accounting for 13.54% of the reviewed 93 papers.

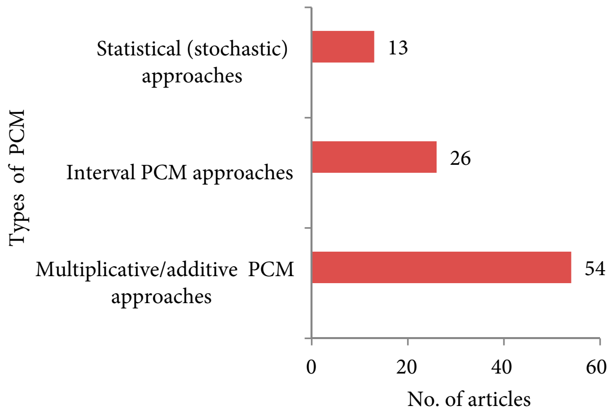

Fig. 1. The distribution of researches on three classes of PCMs

In the multiplicative and additive PCMs, Figure 2 shows that the most prevalent research topics is cardinal inconsistency issue, and fewer researches focused on the ordinal inconsistency and measure scales. For the measure scales in the multiplicative and additive PCMs, Table 2 (see Appendices) shows that the attention has been paid to the size of scale, the verbal scale, numerical scale and relative measure scale. In the studies on consistency indices, some researchers focus on the comparison among the existing consistency indices in order to provide a guideline for selecting appropriate consistency index, some concentrate on the analysis of the properties of the inconsistency indices so as to propose axioms to characterize inconsistency indices, while much attention has been paid to developing new consistency indices for improving the test efficiency, as shown in Table 3 (see Appendices).

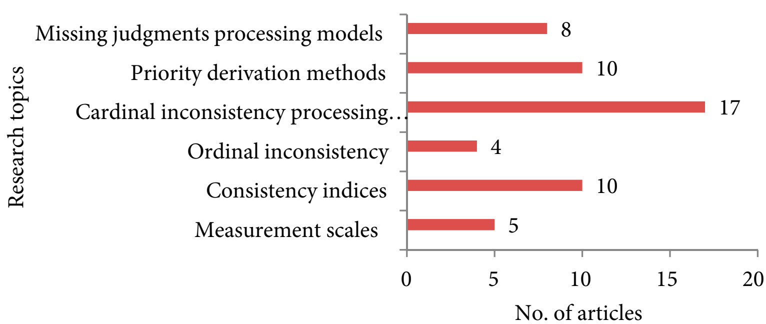

Fig. 2. The distribution of researches on the main developments of multiplicative and additive PCMs

On the inconsistency processing models, the inconsistency issue in a PCM can be grouped into two classes: ordinal inconsistency (intransitivity) and cardinal inconsistency, in which the latter is more popular than the former, as shown in Table 4 (see Appendices). In addition, the graph theoretic approach is usually used to study the ordinal inconsistency. There are various approaches for cardinal inconsistency, including linearization technique and nonlinear optimization approach. Specifically, the consistency ratio can be improved by identifying and modifying the most inconsistent elements, minimizing the distance between the inconsistent PCM and constructed near-consistent PCM, reducing the number of pairwise comparisons and so on.

For the priority derivation methods, Table 5 (see Appendices) shows that some researchers focus on the analysis and comparison among the existing prioritization approaches and provide guidelines for selecting the appropriate derivation method, while others concentrate on developing new derivation methods such as the parametric method, logarithmic transformation based algorithm and cosine maximization method.

The approaches on missing judgments processing models include Multi-Layer Perception (MLP) neural network, a logarithmic goal programming formulation method, linear programming method, graph-theoretic approach, optimal sequencing approach, Frobenius norm based matrix distance minimization approach, connecting path method (CPM) based method and geometric mean induced bias matrix approach, as shown in Table 6 (see Appendices).

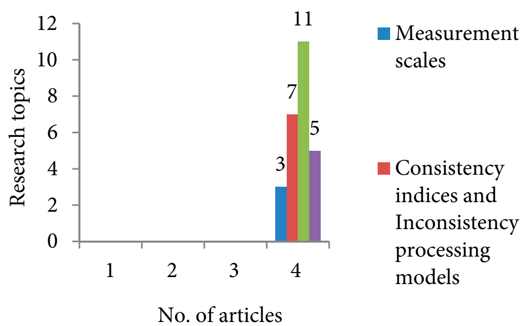

Fig. 3. The distribution of researches on the main developments of interval (fuzzy) PCMs

In the studies on interval (fuzzy) PCM, Figure 3 shows that only three papers focused on measure scales. Much attention still has been paid to the consistency indices, inconsistency processing models, priority derivation methods and missing judgments processing models, in which the most frequently studied topic is the priority derivation methods (11 papers out of 93 papers, 11.83%), followed by consistency indices and inconsistency processing models (7 papers, 7.53%), missing judgments processing models (5 papers, 5.38%) and measure scales (3 papers, 3.23%). It can be seen from Table 7 (see Appendices) that fuzzy techniques were used in the measure scales. Different from the multiplicative and additive PCM, there are more integrated approaches in the interval PCM. For instance, Table 8 (see Appendices) indicates that researchers first proposed a new consistency index, then developed related models to improve the consistency or derive the weights for interval

PCM. The employed approaches contain linear programming model, information granularity, logarithmic Manhattan distance, geometric mean, abelian linearly ordered group (alo-group) and logarithmic least square model etc.

To derive the priority weights for an interval PCM, various techniques have been employed in the reviewed articles, including linear or nonlinear progamming methods, fuzzy data envelopment analysis, approximation and adjustment (AAM) method, Fuzzy Logarithmic Least Square Model, Abelian linearly ordered group (Alo-group) and fuzzy programming method (FPM) etc, as shown in Table 9 (see Appendices).

For the missing judgments estimation problem in interval PCM, we found that two paper focused on consistency, weight prioritization and missing judgments estimation concurrently, and the other three papers paid attention to both consistency and missing judgments estimation procedures. Table 10 also presents that the used techniques include goal programming model, logarithm least squares optimization and mathematical optimization etc.

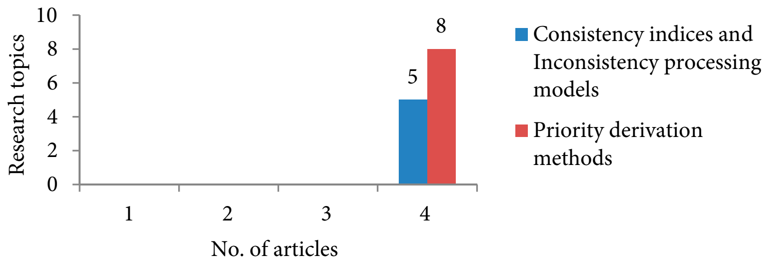

Fig. 4. The distribution of researches on the main developments of statistical approaches

Compared with the previous approaches in multiplicative, additive PCMs and interval PCM, Figure 4 shows that few articles involved in the above mentioned research issues using statistical approach, in which 5 out of ninety-three articles concentrated on the consistency indices and inconsistency processing models using statistical approach such as probability distribution, test hypotheses and maximum likelihood estimation etc (see Appendices, Table 11), while 8 papers focused on the priority derivations. The employed techniques include regression model, statistical analysis, stochastic dominance degree (SDD), PROMETHEE-II, method-of-moments methodology, beta stochastic distribution, non-linear programming mode, stochastic preference analysis (SPA) method, ayesian method, geometric mean, parameterization approximate relation, iterative heuristic rating estimation algorithm and Monte Carlo analytic hierarchy process (MCAHP) etc, as shown in Table 12.

Conclusions

This paper is based on a literature review on the main research topics in the pairwise comparison matrix (PCM) from year 2000 to 2015. First, it was found that various approaches were proposed to deal with the measurement scales, consistency index, inconsistency issues and priority derivation methods either in multiplicative, additive or interval (fuzzy) PCM. The most popular research topic is the inconsistency issue, the employed techniques mainly include linearization technique and nonlinear optimization approach. Second, it was noticed that the priority derivation approaches were paid more attention when the statistical analysis methods are employed. In addition, integrated approaches become popular than single method.

In the era of big data, all the approaches proposed for the above issues in the PCM will face big challenges, especially with the increase of dimension of matrix size, the existing approaches and algorithms for consistency test, inconsistent element identification and modification priority derivation methods are not capable of dealing with these issues for the PCM with large dimensions. The missing judgments in the PCM with large size will be another big challenge in the big data environment. Therefore, new approaches remain to be studied under the big data environment in future.

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APPENDIX

Summarizations of the contents and techniques on the main development of different PCM

Table 2. Summarization of the contents and techniques on Measurement scales in multiplicative and additive PCM

Types of PCM	Topics	Content	Technique	Literature
Multiplicative and additive PCM approaches	Measurement scales	– 3-point scale – Verbal scale – Value changes between different unit measure – Numerical scales – Relative measure	– Mathematical and practical justifications – Transformation, Monte-Carlo simulation – Transformation, Analysis – 2-tuple linguistic modeling – Abelian linearly ordered group (Alo-group)	– Fülöp *et al.* 2010 – Kim *et al.* 2010 – Choo and Wedley 2010 – Dong *et al.* 2013 – Xia *et al.* 2015

Table 3. Summarization of the contents and techniques on consistency indices in multiplicative and additive PCM

Types of PCM	Topics	Content	Technique	Literature
Multiplicative and additive PCM	Consistency indices	– Propose New index: $I_g(A)$ – Propose new index: a maximum eigenvalue threshold – Propose New index: multiplicative geometric consistent index (MGCI) – Propose New index: T-inconsistency indicator – Propose New indices: congruence and dissonance measures – Propose new indices: $I_\Omega(A)$ and $GI_\Omega(A)$ – Propose New index: Triad inconsistencies (ATI) – Investigate the degrees of different indices' agreement – Avoid redundancy – Define properties of the inconsistency indices – Present axioms to characterize inconsistency indices	– Alo-group G algorithm – Mathematical optimization – Mathematical optimization and simulation – Abelian linearly ordered group (alo-group) – Mathematical optimization and Monte-Carlo simulation – Abelian linearly ordered group (alo-group) – Monte-Carlo simulations – Analysis and comparison of ten inconsistency -indices – Analysis and comparison of four inconsistency indices – Analysis and comparison of five inconsistency indices – Mathematical and practical justifications	– Cavallo and D'Apuzzo 2010 – Ergu *et al.* 2014a – Meng and Chen 2015 – Koczkodaj *et al.* 2016 – Siraj *et al.* 2015 – Xia and Chen 2015a – Grzybowski 2016 – Brunelli *et al.* 2013a – Brunelli *et al.* 2013b – Brunelli and Fedrizzi 2015a – Brunelli and Fedrizzi 2015b

Table 4. Summarization of the contents and techniques on inconsistency processing models in multiplicative and additive PCM

Types of PCM	Topics	Content	Technique	Literature
Multiplicative and additive PCM	Ordinal inconsistency	– Eliminate intransitive and contradictory judgments – Rectify the intransitive judgments – Handle the cardinal and ordinal inconsistencies – Propose a tool to check the transitivity	– Graph theory – Heuristic algorithm, Monte-Carlo simulation, graph theory – Mathematical optimization and graph theory – Alo-group	– Kéri 2010 – Siraj *et al.* 2012a – Kou *et al.* 2014a – Cavallo and D'Apuzzo 2015
	Cardinal inconsistency	– Discuss the relationship between the consistency of a PCM and the consistency of the decision maker – Achieve the closest consistent matrix – Identify the most inconsistent elements in the PCM – Improve the efficiency and avoid inconsistency – Find the nearest consistent judgment matrix – Find the closest consistent judgment matrix – Analyze the impact factor of inconsistency – Reduce the dimensionality issue – Rectify the most inconsistent elements of PCMs – Find new modified consistent PCM – Find the minimal number of matrix elements – Adjust the original judgments in a minimum way – Find the minimal distance between the original PCM and the modified PCM – Find the worst triad in every subset to improve the efficient processing of large PCM. – Present theoretical proof and empirical evidence for the distance-based inconsistency – Ensure the consistency property of the PCMs – Study the properties of the consistency and consensus	– Interactive questioning procedures – Linearization technique – Induced bias matrix model – Horizontal, vertical and oblique PC algorithm – Semirings algebra, and optimization model – Frobenius matrix norm and optimization approach – Empirical approach – Binary integer programming – Simulation experiments – Integrated Linear Programming and Eigenvector Methods – Nonlinear mixed-integer optimization approach – Nonlinear programming model – Ant algorithm – Heuristic rating estimation algorithm – Empirical approach and Monte Carlo simulation – bilateral agreement, quasi-arithmetic mean – Consensus optimization model	– Temesi 2010 – Benítez *et al.* 2011 – Ergu *et al.* 2011 – Hsu and Wang 2011 – Hou 2011 – Benítez *et al.* 2012 – Bozóki *et al.* 2013 – Jalao *et al.* 2014a – Ergu *et al.* 2014b – Zhang, H. *et al.* 2014 – Bozóki *et al.* 2014 – Pereira and Costa 2014 – Girsang *et al.* 2014 – Kułakowski *et al.* 2015 – Koczkodaj *et al.* 2015 – Xia and Chen 2015a – Zhang 2016

Table 5. Summarization of the contents and techniques on priority derivation methods in multiplicative and additive PCM

Types of PCM	Topics	Content	Technique	Literature
Multiplicative and additive PCM approaches	Priority derivation methods	– Present two straightforward prioritization methods – Provide the guidelines for selecting the most appropriate prioritization operator – Develop a new parametric prioritization method (PPM) – Propose the ⊙-mean vector – Study the properties of weight extraction methods – Obtain a nearer consistent matrix – Propose a cosine maximization method – Analyze the reliability of the eigenvector method (EM) – Analyze several prioritization methods – Propose two properties of the prioritization procedures	– Arithmetic mean and geometric mean methods – Analysis and comparison of nine prioritization operators and seven measurement criteria – Mathematical programming models – Abelian linearly ordered group (alo-group) – Mathematical optimization – Logarithmic transformation and iterative algorithm – Similarity measure, optimization models – Measurement theory – Goal programming methodologies – Mathematical analysis	– Fedrizzi and Brunelli 2010 – Yuen 2010 – Huo *et al.* 2011 – Cavallo and D'Apuzzo 2011 – Dijkstra 2011 – Lin *et al.* 2013a – Kou and Lin 2014 – Tomashevskii 2015 – Jablonsky 2015 – Kułakowski 2015b

Table 6. Summarization of the contents and techniques on missing judgments processing models in multiplicative and additive PCM

Type of PCM	Topics	Content	Technique	Literature
Multiplicative and additive PCM approaches	Missing judgments processing models	– Estimate the missing judgments in an incomplete PCM and improve its consistency simultaneously – Retrieve a group priority vector of the considered alternatives – Propose a method based on the generation of all possible preferences from a set of judgments – Elicit decision maker's preferences in large-dimensional decision problems – Propose properties of completion of PCM	– Multi-Layer Perception (MLP) neural network – Connecting path method (CPM), Hadamard product – Geometric mean induced bias matrix – Similarity function and parametric compromise function, Logarithmic goal programming method – Graph theory and Monte-Carlo simulation – Scoring function, and iterative algorithm – Graph theory, linearization and convex minimization	– Gomez-Ruiz *et al.* 2010 – Chen *et al.* 2015 – Ergu *et al.* 2016 – Dopazo and Ruiz-Tagle 2011 – Siraj *et al.* 2012b – Fedrizzi and Giove 2013 – Benítez *et al.* 2014

Table 7. Summarization of the contents and techniques on measurement scales in Interval PCM

Type of PCM	Topics	Content	Technique	Literature
Interval PCM	Measurement scales	– Analyze individual numerical scale – Propose a new preference scale – Analyze interval numerical scale	– 2-tuple fuzzy linguistic model – Modified interval-valued intuitionistic fuzzy weighted averaging – Consistency-driven methodology	– Dong *et al.* 2011 – Abdullah and Najib 2014 – Dong and Herrera-Viedma 2015

Table 8. Summarization of the contents and techniques on consistency indices and inconsistency processing models in Interval PCM

Type of PCM	Topics	Content	Technique	Literature
Interval PCM	Consistency indices and Inconsistency processing models	– Define a consistency index for the interval matrix and derive relative weights – Investigate the consistency and the consensus of the individual PCMs – Propose a new consistency index ICI ($\tilde{V}$) – propose a geometric mean based uncertainty index – Propose consistency indices for triangular fuzzy numbers (PCFN) matrices – Propose a geometric mean based indeterminacy index – Investigate consistency and priority derivation of Triangular Fuzzy Preference Relation (TFPR)	– Multi-objective linear optimization – Information granularity – Logarithmic Manhattan distance, linear programming – Optimization models – Abelian linearly ordered group (alo-group) – Mathematical approach – Geometric mean, logarithmic least square model	– Conde and de la Paz Rivera Pérez 2010 – Pedrycz and Song 2011 – Dong *et al.* 2014 – Wang 2015a – Ramík 2015 – Li *et al.* 2016 – Wang and Chen 2014

Table 9. Summarization of the contents and techniques on priority derivation methods in Interval PCM

Type of PCM	Topics	Content	Technique	Literature
Interval PCM	Priority derivation methods	– Derive the weight from intervals multiplicative preference relations – Propose a new approach for ranking the alternatives – Generate and ranking multiplicative weights – Derive the fuzzy weights of criteria – Derive the crisp priority vector – Derive the final priority interval weights for both consistent and inconsistent interval PCM – Define concepts on the reciprocity and consistency as well as priority vector – Derive the priority vector from fuzzy PCMs – Derive weights from the PCM with intuitionistic fuzzy numbers (IFNs) – Derive the interval priority vector from the interval preference relations	– Linear programming models, continuous ordered weighted averaging (geometric) operator – Fuzzy data envelopment analysis – Approximation and adjustment (AAM) method – Goal programming method – Modified Fuzzy Logarithmic Least Square Model – Linear or nonlinear programming models – Abelian linearly ordered group (Alo-group) – New fuzzy programming method (NFPM) – Nonlinear optimization problem – Eigenvalue method and the row geometric mean method	– Xu and Cai 2012 – Mirhedayatian *et al.* 2013 – Lin *et al.* 2013 – Izadikhah 2013 – Mohtashami 2014 – Zhang, F. *et al.* 2014 – Ramík 2015 – Chen and Xu 2015 – Dutta and Guha 2015 – Meng *et al.* 2015

Table 10. Summarization of the contents and techniques on missing judgments processing models in Interval PCM

Type of PCM	Topics	Content	Technique	Literature
Interval PCM	Missing judgments processing models	– Complement the acceptable missing values and obtain the priority vector from incomplete IMPR – Estimate missing values and obtain the priority vector from incomplete IMPR – Estimate the missing values in the fuzzy PCMs – Construct a complete additive consistent interval preference relation – Repair incomplete hesitant fuzzy preference relations	– Goal programming model, interval weighted geometric averaging (IWGA) operator – Logarithm least squares optimization and completion methods – Analysis of the relations between transitivity and consistency of fuzzy PCMs and multiplicative PCMs – Mathematical optimization methods – Mathematical optimization methods	– Liu *et al.* 2012 – Wang and Chen 2014 – Ramík 2014 – Xu *et al.* 2014 – Zhang *et al.* 2015

Table 11. Summarization of the contents and techniques on Consistency indices and inconsistency processing models in Statistical (stochastic) approaches

Approaches	Topics	Content	Technique	Literature
Statistical (stochastic) approaches	Consistency indices and Inconsistency processing models	– Search the maximum possible deviation in the PCM – Solve the stochastic multiple criteria decision making (SMCDM) problem – Define the uncertainty indices for intervals – Develop an improved statistical approach for consistency test of the PCM – Propose a new statistical approach to deal with consistency issue	– Genetic algorithm – Dominance degree matrix, probability distributions, and PROMETHEE II – Entropy in probability, sum or maximum of widths – Test hypotheses and maximum likelihood estimation – Hypothesis test and the random consistency index	– Tsyganok 2010 – Liu *et al.* 2011 – Entani and Sugihara 2012 – Lin *et al.* 2013b – Lin *et al.* 2014

Table 12. Summarization of the contents and techniques on priority derivation methods in Interval PCM

Approaches	Topics	Content	Technique	Literature
Statistical (stochastic) approaches	Priority derivation methods	– Estimate the priority weights – Derive the priority weights for ranking alternatives – Fit the varying stochastic preferences of the DM into beta stochastic pairwise comparisons – Stochastic preference analysis in numerical preference relations – improve the individual PCMs – Define a geometric mean based uncertainty index – Deal with the situation when exact estimations for some concepts (stimulus) CK are a priori known and fixed – Compare the results of AHP with Monte Carlo analytic hierarchy process (MCAHP)	– Statistical analysis,regression model – Stochastic dominance degree (SDD), PROMETHEE-II – Method-of-moments methodology – Stochastic preference analysis (SPA) method, Monte Carlo simulation – Bayesian revision method, lognormal errors – Parameterization approximate relation – New iterative heuristic rating estimation algorithm – Simulation approach, statistical analysis	– Bernasconi *et al.* 2010 – Zhang *et al.* 2010 – Jalao *et al.* 2014b – Zhu and Xu 2014 – Lin and Kou 2015 – Wang 2015b – Kułakowski 2015a – Yaraghi *et al.* 2015)

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