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DETERMINING OF VEHICLE CAPACITY BASED ON A LOT SIZE OF GOODS

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Abstract. The size of lots of goods, vehicle type and capacity as well as the method of transportation should be properly matched. The coordination of these key parameters could reduce freight transportation costs. This can also help to save time. The proper choice of the type of a vehicle to carry the particular cargo could reduce the wear of transport facility and ensure freight safety on the road. The above measures would result in saving of resources.

Keywords: vehicle, cargo-carrying capacity, type of vehicle, transportation.

1. Introduction

Prior and after the integration of Lithuania into the European Union the priority issue is rational allocation of resources obtained from various financing organizations in the area of transport. This problem is of particular importance to our country possessing rather old fleet of vehicles, which requires large investments. The expenses can be reduced to some extent by the proper choice of the vehicles and the ways of transportation in respect to cargo lot sizing. There are, however, some objective factors preventing to achieve this. The failure is often caused by subjective reasons or poor management resulting in even higher expenses. Given an old fleet of vehicles, it is still possible to find an effective solution based on mathematical, statistical and probabilistic approaches [1–5]. The above solutions would allow us to save time and money.

The use of mathematical methods enables the optimization of complicated processes of transportation and transport policy [6].

2. Determining of the structure of the fleet of optimal capacity vehicles

The structure of the fleet of vehicles based on their cargo-carrying capacity should meet the requirements of transporting goods in lots of various size.

Let cargo-carrying capacity of a vehicle be represented by a series $q_1, q_2, \ldots, q_j, \ldots, q_m$. In addition, size distribution of the lots of goods is known. The probability of a lot of goods which would require the vehicle of q_j $(j = 1, 2, \ldots, m)$ carrying capacity for transportation is as follows:

$$p_j = \begin{cases} \int_0^{(q \cdot r)_j} f(x)\,dx, & j = 1; \\ \int_{(q \cdot r)_{j-1}}^{(q \cdot r)_j} f(x)\,dx, & j > 1, \end{cases} \tag{2.1}$$

here $f(x)$ – distribution density of lot sizes; γ – carrying capacity coefficient.

The probability of occurrence of a lot of goods requiring q_m capacity vehicle which would transport a lot of goods by i hauls $(i = 1, 2, \ldots)$ would be:

$$p_{m,i} = \begin{cases} \int_{(q \cdot r)_{m-1}}^{(q \cdot r)_m} f(x)\,dx, & j = 1; \\ \int_{(i-1)(q \cdot r)_m}^{i(q \cdot r)_m} f(x)\,dx, & i > 1. \end{cases} \tag{2.2}$$

A number of vehicles j of the type $(j = 1, 2, \ldots, m)$ needed is as follows:

$$A_{ej} = \frac{\bar{N}_{v.r.} p_j}{T_{nj}} \left(\frac{l_{g.ej}}{V_{tj} \beta_j} + t_{npj} \right), \tag{2.3}$$

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here $\overline{N}_{v.r.}$ – average number of requests for goods transportation per 24 hours; T_{nj} – working hours of vehicle per 24 h, h; $l_{g.ej}$ – a run of loaded vehicle, km; V_{tj} – vehicle speed, km/h; β_j – coefficient of the run; t_{npj} – loading time of a haul, h.

A required number of q_m capacity vehicles:

$$A_m = \frac{\overline{N}_{v.r.} \sum_{i=1}^{\infty} i p_{m,i}}{T_{nm}} \left(\frac{l_{g.em}}{V_{tm}\beta_m} + t_{npm} \right). \quad (2.4)$$

Total number of vehicles:

$$A_e = \sum_{j=1}^{m} A_{ej} = \overline{N}_{v.r.} \left[\sum_{j=1}^{m-1} \frac{p_j}{T_{nj}} \left(\frac{l_{g.ej}}{V_{tj}\beta_j} + t_{npj} \right) + \frac{\sum_{i=1}^{\infty} i p_{m,i}}{T_{nm}} \left(\frac{l_{g.em}}{V_{tm}\beta_m} + t_{npm} \right) \right]. \quad (2.5)$$

By dividing the left and the right sides of the equations (2.3) and (2.5), we get:

$$\frac{A_{ej}}{A_e} = \frac{p_j}{T_{nj} B} \left(\frac{l_{g.ej}}{V_{tj}\beta_j} + t_{npj} \right), \ j = 1, 2, \ldots, m-1. \quad (2.6)$$

Similarly, from the equations (2.4) and (2.5) we obtain:

$$\frac{A_m}{A_e} = \frac{\sum_{i=1}^{\infty} i p_{m,i}}{T_{nj} B} \left(\frac{l_{g.ej}}{V_{tj}\beta_j} + t_{npj} \right) \quad (2.7)$$

and from (2.6) and (2.7) we get:

$$B = \frac{A_e}{\overline{N}_{v.r}} = \sum_{j=1}^{m-1} \frac{p_j}{T_{nj}} \left(\frac{l_{g.ej}}{V_{tj}\beta_j} + t_{npj} \right) + \frac{\sum_{i=1}^{\infty} i p_{m,i}}{T_{nm}} \left(\frac{l_{g.em}}{V_{tm}\beta_{mj}} + t_{npm} \right). \quad (2.8)$$

If $T_{nj} = T_{nm} = T_n$, then we should calculate:

$$T_n B = \sum_{j=1}^{m-1} p_j \left(\frac{l_{g.ej}}{V_{tj}\beta_j} + t_{npj} \right) + \left(\frac{l_{g.em}}{V_{tm}\beta_m} + t_{npm} \right) \sum_{i=1}^{\infty} i p_{m,i}. \quad (2.9)$$

Therefore, to determine the probability of requests for transporting goods by various capacity vehicles means to find the type of size distribution of lots and the average output of the above vehicles per 24 hours.

Exponential distribution of lot sizes can be expressed in the following way:

$$f(x) = \frac{1}{\overline{g}} e^{-\frac{x}{\overline{g}}}, \quad (2.10)$$

here $\overline{g}$ – average lot size of goods, t.

$$p_1 = \frac{1}{\overline{g}} \int_0^{(q \cdot r)_1} e^{-\frac{x}{\overline{g}}} dx = 1 - e^{-\frac{(q \cdot r)_1}{\overline{g}}}, \quad (2.11)$$

$$p_j = \frac{1}{\overline{g}} \int_{(q \cdot r)_{j-1}}^{(q \cdot r)_j} e^{-\frac{x}{\overline{g}}} dx = e^{-\frac{(q \cdot r)_{j-1}}{\overline{g}}} - e^{-\frac{(q \cdot r)_j}{\overline{g}}}, \quad (2.12)$$

$$p_{m,i} = \frac{1}{\overline{g}} \int_{(i-1)(q \cdot r)_m}^{i(q \cdot r)_m} e^{-\frac{x}{\overline{g}}} dx = e^{-\frac{(i-1)(q \cdot r)_m}{\overline{g}}} - e^{-\frac{i(q \cdot r)_m}{\overline{g}}}. \quad (2.13)$$

If the lot sizes is distributed according to the normal law, the probability of a random value q to be in the interval [$(q \cdot r)_{j-1}$; $(q \cdot r)_j$] may be found in the following way:

$$p_j = P\{(q \cdot r)_{j-1} < q < (q \cdot r)_j\} = \Phi^* \left[\frac{(q \cdot r)_j - \overline{g}}{\sigma_g} \right] - \left[\frac{(q \cdot r)_{j-1} - \overline{g}}{\sigma_g} \right], \quad (2.14)$$

here σ_g – mean square deviation of the random value.

$$p_{m,i} = \begin{cases} \Phi^* \left[\frac{(q \cdot r)_m - \overline{g}}{\sigma_g} \right] - \Phi^* \left[\frac{(q \cdot r)_{m-1} - \overline{g}}{\sigma_g} \right], & i = 1; \\ \Phi^* \left[\frac{i(q \cdot r)_m - \overline{g}}{\sigma_g} \right] - \Phi^* \left[\frac{(i-1)(q \cdot r)_m - \overline{g}}{\sigma_g} \right], & i > 1. \end{cases} \quad (2.15)$$

In some cases, transporters and shippers relate the lot size of goods to cargo-carrying capacity of a vehicle. Then an average lot size of goods to be transported will be:

$$\overline{g} = \sum_{j=1}^{m-1} p_j (q \cdot r)_j + (q \cdot r)_m \sum_{i=1}^{\infty} i p_{m,i}, \quad (2.16)$$

here $(q \cdot r)_j, (q \cdot r)_m$ – the largest vehicle capacities based on vehicle body capacity and the kind of transported goods.

An average lot size of goods carried in a haul:

$$\overline{g}_e = \sum_{j=1}^{m-1} p_j (q \cdot r)_j + (q \cdot r)_m \sum_{i=1}^{\infty} p_{m,i}. \quad (2.17)$$

An average vehicle cargo-carrying capacity calculated per haul:

$$\bar{q}_e = \sum_{j=1}^{m-1} p_j q_j + q_m \sum_{i=1}^{\infty} p_{m,i} . \quad (2.18)$$

An average value of the static coefficient of the utilized vehicle fleet capacity:

$$\gamma_{st} = \frac{\bar{g}_e}{\bar{q}_e} = \frac{\sum_{j=1}^{m-1} p_j (q \cdot r)_j + (q \cdot r)_m \sum_{i=1}^{\infty} p_{m,i}}{\sum_{j=1}^{m-1} p_j q_j + q_m \sum_{i=1}^{\infty} p_{m,i}} . \quad (2.19)$$

A number of hauls made by the vehicles of the fleet in a considered period:

$$n_e = \frac{P}{\bar{q}_e \bar{\gamma}_{st}} , \quad (2.20)$$

here P – total volume of transported goods, tons.

A number of hauls made by j – type vehicles:

$$n_{ej} = p_j n_e, \quad j = 1, 2, \ldots, m-1 , \quad (2.21)$$

and by the largest capacity vehicles:

$$n_{em} = n_e \sum_{i=1}^{\infty} p_{m,i} = n_e - \sum_{j=1}^{m-1} n_{ej} . \quad (2.22)$$

Total volume of goods carried by q_j capacity vehicles:

$$P_j = n_{ej} (q \cdot r)_j , \quad j = 1, 2, \ldots, m . \quad (2.23)$$

The required number of q_j capacity vehicles:

$$\bar{A}_j = \frac{P_j}{D \alpha_j P_{\text{par.}j}} , \quad j = 1, 2, \ldots, m , \quad (2.24)$$

here $P_{\text{par.}j}$ – vehicle output per 24 hours; α – number (coefficient) of vehicles sent; D – number of days in a considered period.

$$P_{\text{par.}j} = \frac{\upsilon_{tj} \beta_j q_j \gamma_{stj} T_{nj}}{l_{g.ej} + \upsilon_{tj} \beta_j t_{npj}} . \quad (2.25)$$

3. Proper lot sizing and the choice of a transportation method

In the transportation of goods based on the system of terminals and long-term contracts between suppliers and customers, optimal lot sizing and choice of transportation technique make an important problem [7–8]. Transporting large lots can reduce the haulage costs, however, in this case, storage costs are higher. Transporting small lots of goods, the situation is opposite.

In the present paper methods of optimal lot sizing and proper choice of transportation technique are suggested. The optimality criterion is based on general costs of storage and transportation.

Let us analyze three methods of transportation:

1. The transportation companies carry goods in large lots according to long-term contracts, loading trucks and trailers to their full capacity, or using several vehicles.
2. Goods are delivered in medium-size lots partially loading vehicles or trailers. Firms carrying goods in this way combine them into large lots at the terminals.
3. Goods are delivered in small lots (up to 23 kg) being formed at some small areas.

Let us denote by: C – overall costs; C_T – costs of transportation; C_1 – cargo storage costs, then:

$$C = C_T + C_1 . \quad (3.1)$$

Storage costs are found in the following way:

$$C_1 = P[(V/U + T) + T], \quad (3.2)$$

here T – time of cargo transportation (weeks); U – rate of supply and consumption (kg/week); P – freight cost (Lt/kg); V – costs of freight storage (Lt/kg per week).

The costs of transporting 1 kg of cargo are obtained as follows:

– by transportation method 1:

$$C_T = F/V_1, \quad V \le S ; \quad (3.3)$$

– by methods 2 and method 3 of transportation:

$$C_T = F/V + W , \quad (3.4)$$

here F – charge fixed for transporting 1-st lot of goods; S – vehicle capacity (kg); W – cost of transporting 1 kg of cargo for any lot size.

From (3.1), (3.2), (3.3), (3.4) one can see that overall costs will be minimum if the lot size is as follows:

$$V* = \min\left(\sqrt{FU/PR\,S}\right), \quad (3.5)$$

while the costs of transporting 1 kg of cargo are:

$$C* = \begin{cases} 2\sqrt{FPR/U} + W + PRT, \\ PRS/U + F/S + W + PRT & \text{in the opposite case.} \end{cases} \quad (3.6)$$

In Fig 1, common relationships between overall costs of freight transportation and storage and supply rate are given.

In Fig 2, a solid line shows the relationship between the optimal lot size and supply rate.

One can see that for the low supply rate method 3 of transportation is optimal, while for medium supply rate transportation method 2 is efficient, and for high supply rate method 3 is effective. The relationship between the lot size of goods and supply rate is not a continuous function. The relationships obtained allow for the optimization of transportation for various initial data.

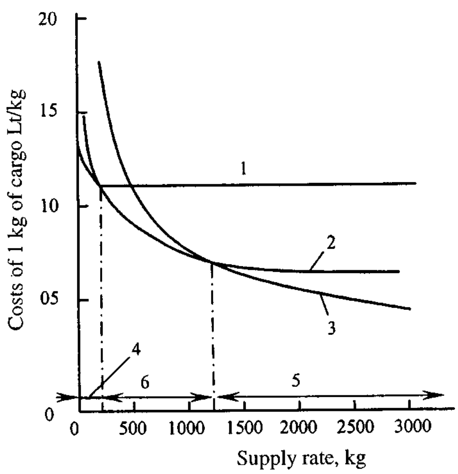

Fig 1. The dependence of general costs of transporting one kg of cargo on supply rate by using various freight transportation methods: 1 – transportation method 3; 2 – transportation method 2; 3 – transportation method 1; 4, 5, 6 – supply rate, when methods 3, 2 and 1 are optimal

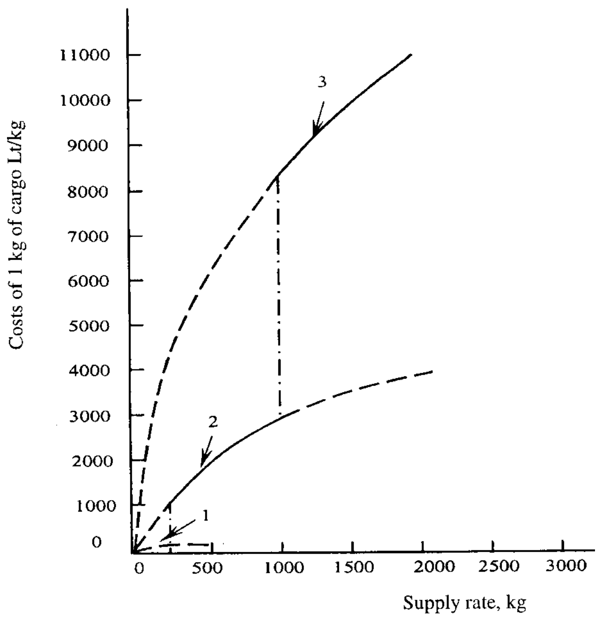

Fig 2. Relationship between the optimal lot size and supply rate: 1, 2 and 3 – 1, 2 and 3 transportation methods

4. Conclusions

1. To determine the optimal structure of the fleet of vehicles and transportation technique lots and amount of goods should be considered in terms of time. Transportation requests and lots of goods are random values. Methods of mathematical statistics are the most effective in their analysis in respect of time.

2. Methods allowing the efficient sizing of lots of goods and transportation technique are suggested. They are based on the criterion of general storage and transportation costs.

3. Based on the methods suggested, optimal vehicle fleet structure may be determined taking into account carrying capacity of vehicles and their average number.

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